Selecting Sample Trees to Estimate

Douglas-fir Basal Area Increment Distribution

Among 2-inch Diameter Classes

by

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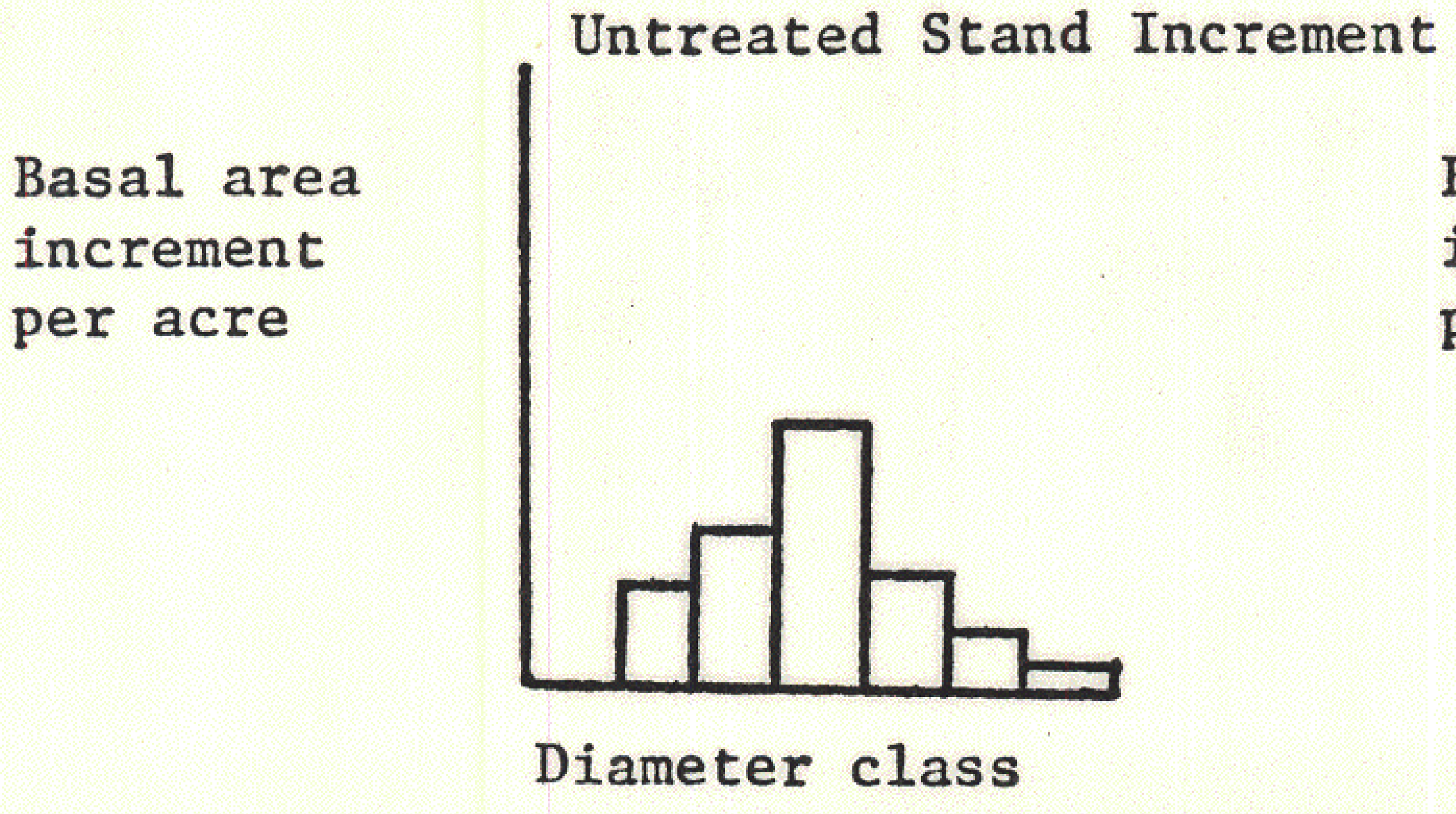
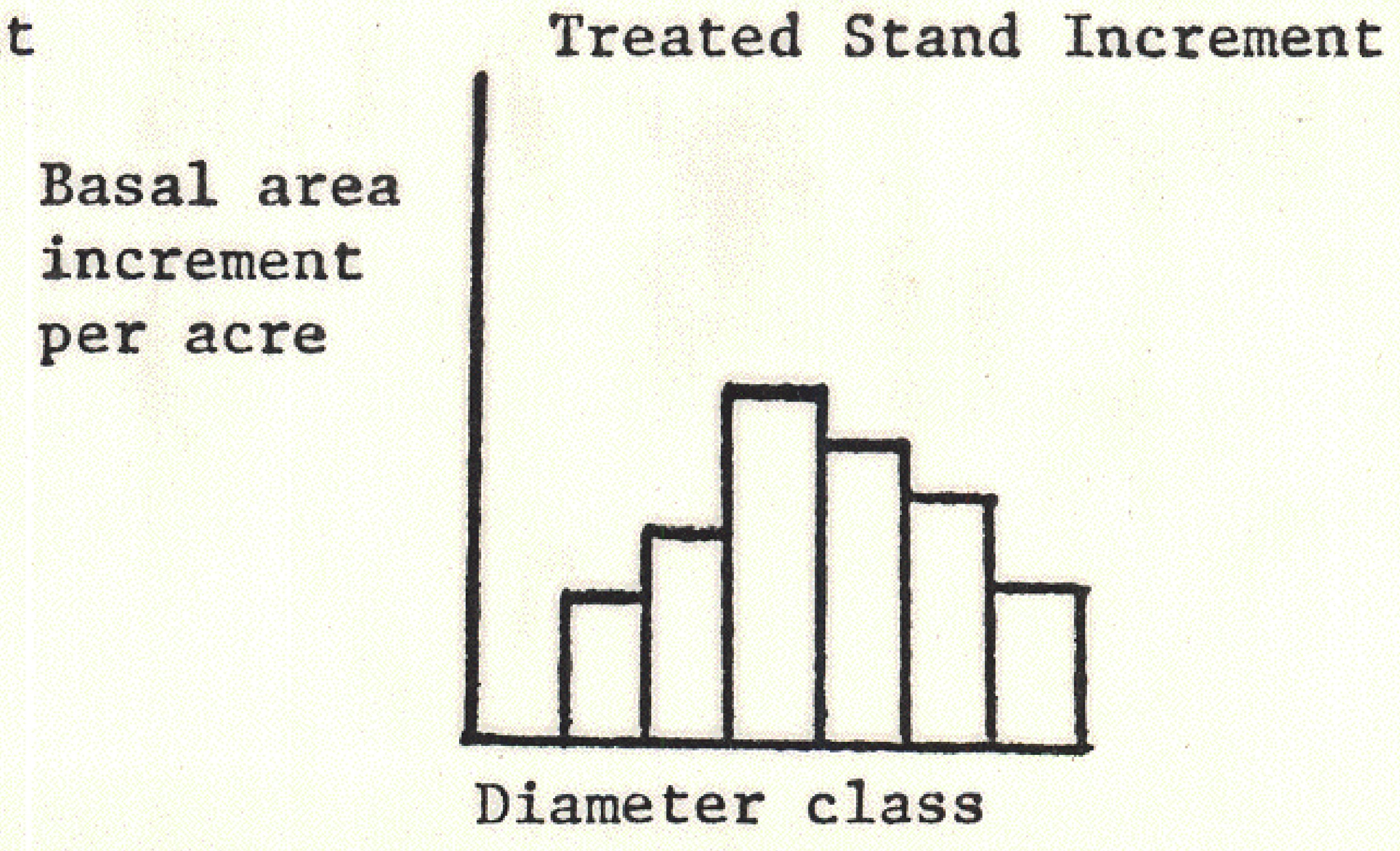
Introduction

Forest stand growth is assessed in terms of (a) total basal area increment per acre, (b) total volume increment per acre, and (c) total increment per acre distributed among diameter classes within the stand. Both former terms express wood production only on a total area basis. The latter expression also defines the contribution of each diameter class to that total area production.

Silvicultural stand treatments such as forest fertilization, forest irrigation, and stocking control are evaluated in these forest growth terms. The contribution of increment from each diameter class provides an expression for effects of these silvicultural stand treatments within the stand structure. Increment distribution analysis also allows comparison of these treatment effects in relation to tree size as well as in relation to total area production. Such a comparison is graphically shown below:





This basal area increment distribution by diameter class (BG_i, where i refers to the i^{th} diameter class) is a function of diameter increment from all trees in the i^{th} class added together. In equation form:

$$BG_i = .005454154 \sum_{j=1}^{N_i} (D_{2j}^2 - D_{1j}^2)$$

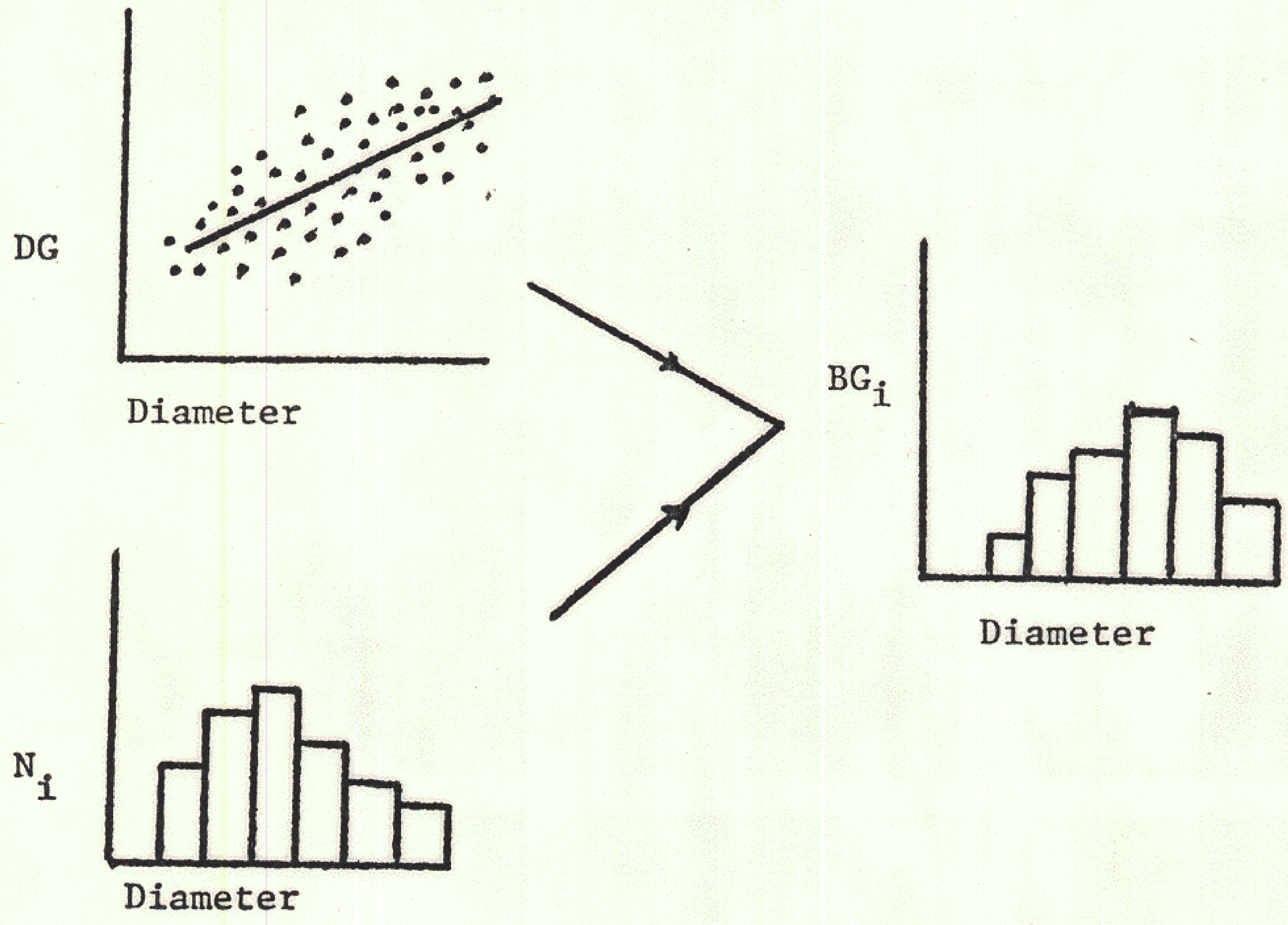
where Ni = number of trees in the i^{th} diameter class

D_{2j} = Final diameter of the j^{th} tree in the i^{th} class

D_{1j} = Initial diameter of the j^{th} tree in the i^{th} class

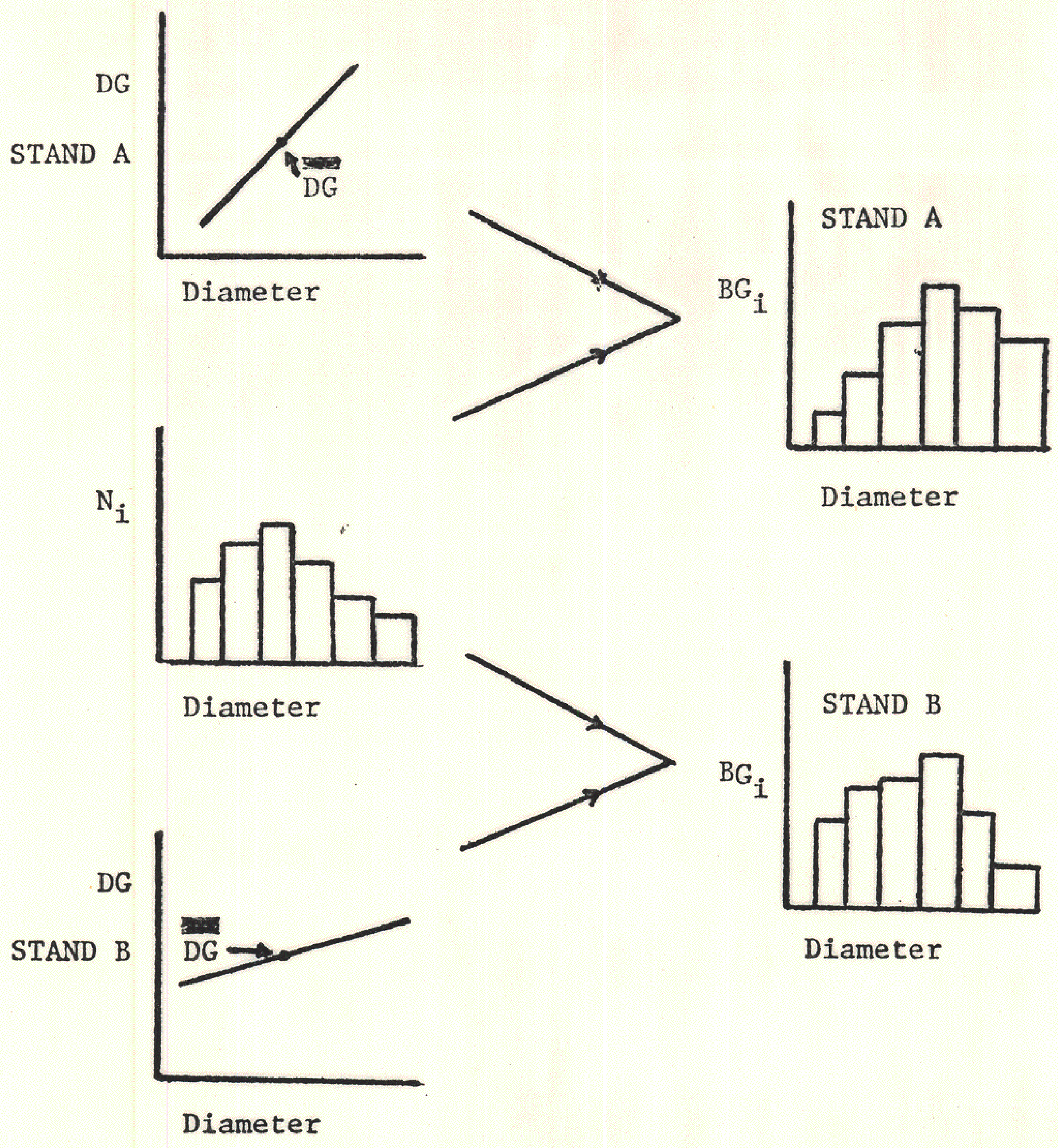
BG_i = Basal area increment for the i^{th} class

This dynamic relationship of diameter increment (DG) and stem frequency (N_i) to BG_i is further illustrated below:



The component $\overline{DG}_i$, the mean diameter increment by diameter class, can be compared for treated and untreated conditions to assess effects of treatment on diameter increment.

The estimate of the slope of the diameter increment trend is important because the slope, b, determines how the basal area increment is distributed among diameter classes. A graph of two stands having increment trends with the same overall mean diameter increment, $\overline{DG}$, but different slopes will illustrate this important concept.



The total basal area increment per acre is the same for both stands because $\overline{DG}$ is the same and both trends are applied to the same stem frequency distribution (N_i). But the basal area increment by diameter class is distributed quite differently in stand A than in stand B because the slopes of the increment trends are different. The steeper the slope, the more the basal area increment distribution is skewed to the larger diameter classes (Stand A). Conversely, the flatter the slope, the more B_i is skewed to the smaller diameter classes (Stand B).

To better understand how stand treatment affects distribution of basal area increment in Douglas-fir (BG_i), more study is needed to examine growth trends (DG) on natural, <u>untreated</u> stands of Douglas-fir. The growth data analyst must know how much natural variation in diameter increment to expect from untreated stands. This natural variation has not been adequately identified for most stand conditions.

Data on the components of basal area increment distribution, DG and N_i, are conventionally derived from remeasurement of <u>all</u> trees on permanent, fixed area sample plots located to represent treated and untreated stands. This is often a costly and time consuming operation.

To reduce this time and cost, several options are open to the experimenter. He may i) reduce plot size and remeasure all trees or ii) maintain original plot size but remeasure only a fraction of all trees. If he chooses the latter option, he can either sample over the whole range of diameters or sample only selected portions of the diameter range.

OBJECTIVE

The objective of this study is to investigate sampling plans by which fewer than all trees on the plot are measured and compare the relative merit of these plans according to their estimate of:

1) Total basal area increment
2) Total basal area increment distribution, and
3) Variance of increment

The natural variation of diameter increment will be examined for presence of trend relative to diameter, since this can affect the proposed sampling plans and estimation procedures.

Hohenadl (1939) was among the first foresters to develop stand volume estimation methods based on a subsample of all trees on the plot. He estimated the volume in two mean trees located one standard deviation below and one standard deviation above the mean diameter of the stand. Total volume in the stand was then estimated from

$$V = N \frac{V_1 + V_2}{2}$$

where N = total number of stems

V_1 = volume of tree below mean diameter

V_2 = volume of tree above mean diameter

V = total stand volume

Another subsampling method was used by Stage (1960) to compute total growth from increment cores with point sampling. His procedure of selecting increment trees with a prism concentrates sampling effort according to growth potential (tree size) rather than tree frequency. Estimates of the mean basal area per acre and the mean increment ratio are derived rather than a direct estimate of basal area increment. This method was not designed to give estimates of the distribution of basal area increment.

Another forester, Emrović (1967), derived mathematically a method of choosing increment sample trees. He outlined a method of selecting sample trees so that the standard deviation of the stand volume increment calculated from increment cores of the sample trees is at a minimum. Emrović further derived a relationship showing that the number of sample trees in any diameter class is proportional to the product of number of trees in the diameter class and the mean volume of that class. This is based on the assumption that variance of volume increment is proportional to tree size.

Another example is an empirical approach to choosing increment sample trees based on trees having the stand mean diameter. Guminski (1969) demonstrated that trees in the stand mean diameter class are also those trees on which mean diameter increment occur. The data were from 855 spruce trees and 717 silver fir trees in Poland. He reported no mathematical models and no tests of hypotheses concerning general growth trends.

Statisticians who have investigated methods of estimating distribution of increment are also among the minority. Two who have worked on a related subject are Wald (1940) and Bartlett (1949). These statisticians were concerned with fitting a straight line when both x and y are subject to error.

Wald used a sampling procedure similar to that of Hohenadl - choosing observations from only the two extreme groups of the data range. From these samples, Wald derived confidence interval formula for the regression estimates, a and b.

Bartlett modified Wald's procedure by sampling from non-overlapping strata (when considered in the x-direction) using three groups, two from the extremes of the range of x and one toward the center of the range of x.

He divided the n plotted points into three groups, the equal number k in the two extreme groups chosen to be as near 1/3 n as possible. The join of the mean coordinates $\overline{x}_1$, $\overline{y}_1$ and $\overline{x}_3$, $\overline{y}_3$ for the two extreme groups is used to determine the slope, b; the location of the line is determined by the overall means $\overline{x}$, $\overline{y}$ as in conventional least squares fitting. He used this sampling design to develop a method of determining the variance of b, s_b^2. His formula for s_b^2 follows:

$$s_b^2 = \frac{2\,\hat{\sigma}^2}{k\,(\bar{x}_3 - \bar{x}_1)^2}$$

$\hat{\sigma}^2$ is the estimate of population variance of y.

From the formula for s_b^2 it is evident that the further apart the two extreme group means, the less the variance of the slope.

Although the few works outlined above contain most of the sparse research on this subject, increment distribution analysis is in fact a critical ingredient to stand treatment research. The distribution of increment altered by silvicultural stand treatment is one measure of the worth of that stand treatment. A major objective of stocking control for example, is to redistribute growth potential of the stand to optimum advantage (Staebler, 1961).

Estimates of increment distribution allow comparison of stand treatment effects in relation to tree size as well as comparison of total wood produced. This study compares sampling schemes using reduced numbers of increment sample trees in an attempt to estimate diameter increment trends without unduly sacrificing precision of growth estimates, thus reducing time and cost in data collection and analysis.

METHODOLOGY

There are two main phases of this study. They are:

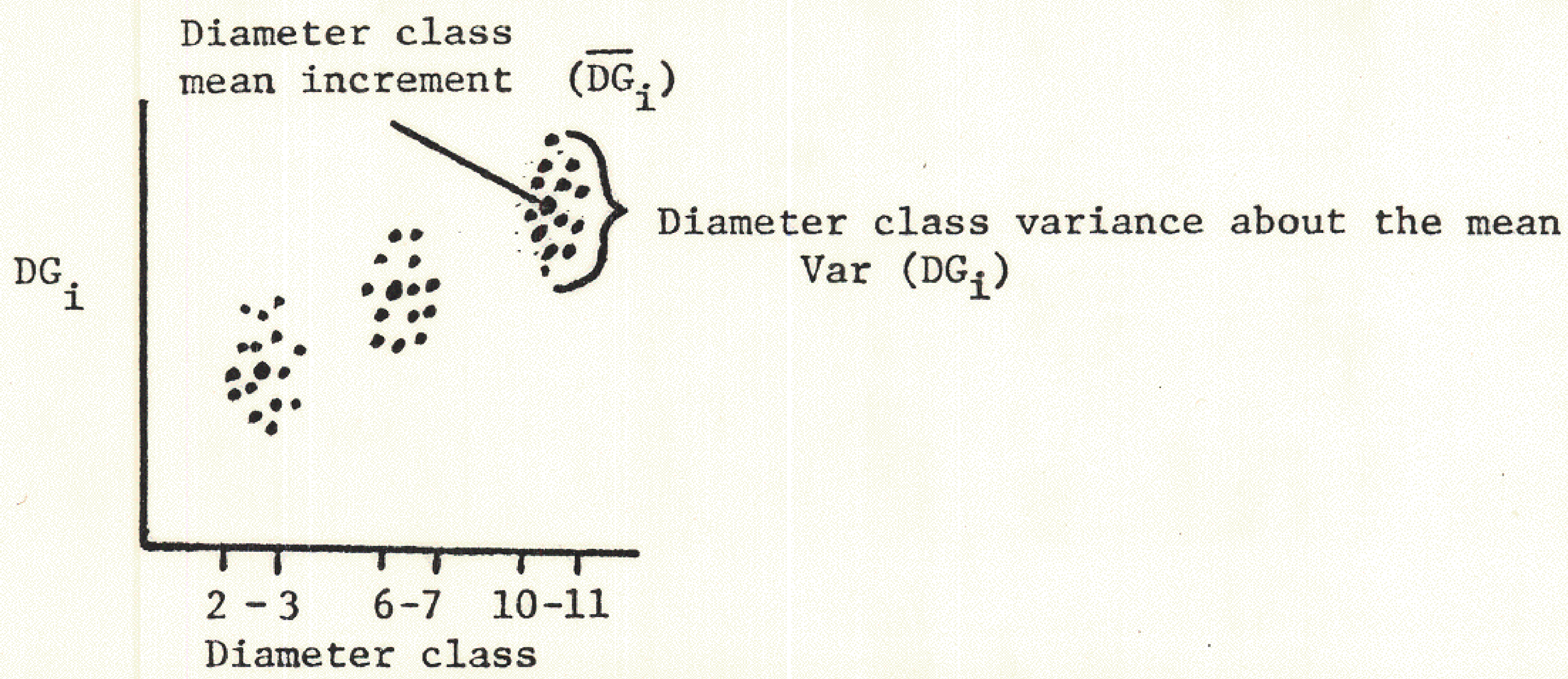
I. Test assumptions underlying the estimation procedure

II. Conduct sampling experiments to test several alternate sampling plans

Phase I: Test assumptions underlying the estimation procedure

The sampling method used to obtain precise estimates of population means depends on how the variance of the sample population is distributed. For example, if variance is uniform in all parts of the population, then sampling might be conducted uniformly throughout the population. If variance is not uniform, as is often the case, the sampling design becomes critical if the sample is to represent the population. One such method is stratified random sampling.

In this study the aim of sampling is to estimate the mean diameter increment $\overline{DG}_i$ within each diameter class. Therefore, the variance about that mean, Var (DG_i), and the distribution of the variance relative to tree diameter throughout the population is of concern. The following graphical representation of this variance may be helpful:



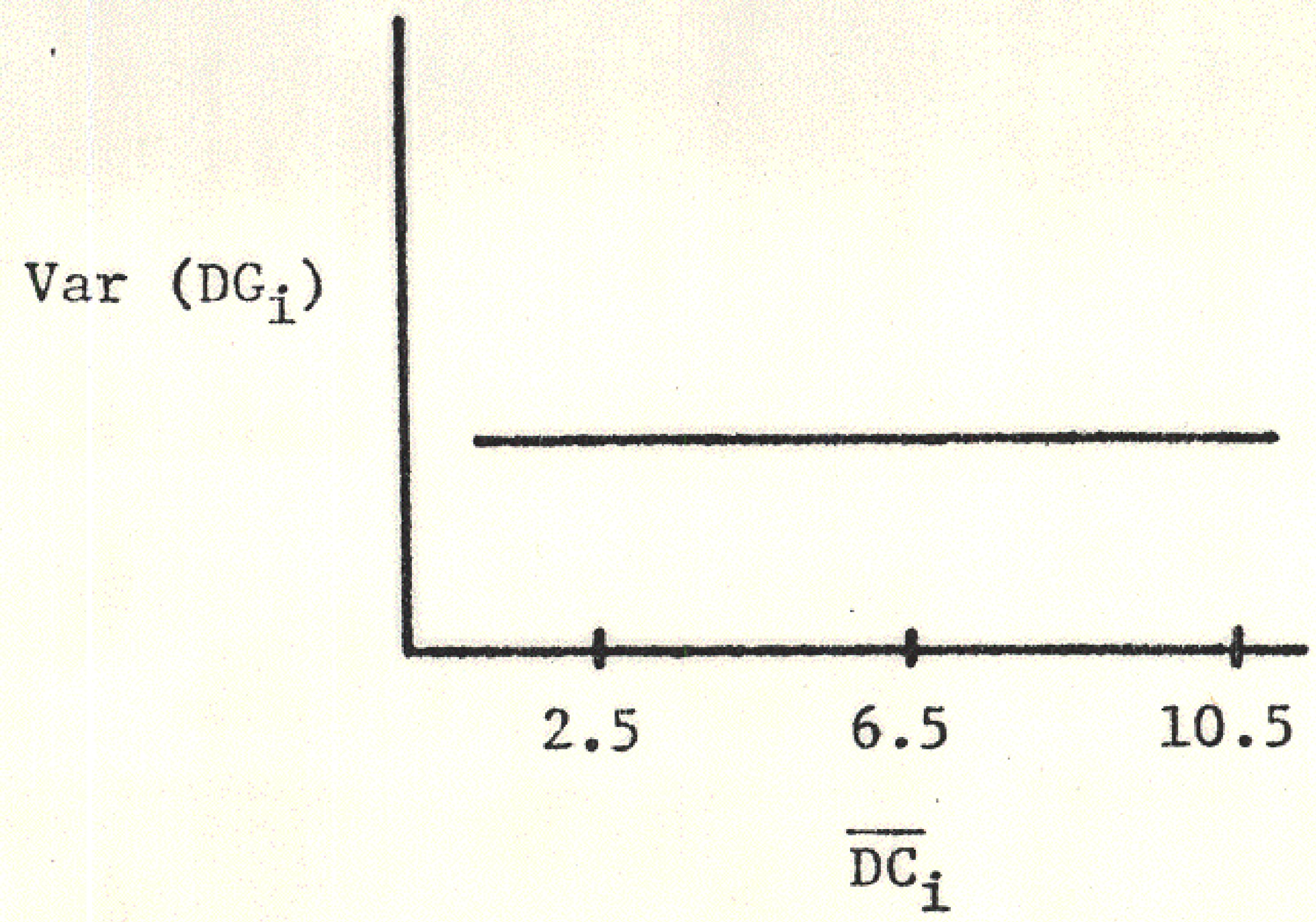
The distribution of this variance is investigated by testing the null hypothesis that Var (DG_i) is independent of tree size. In more formal terms, null hypothesis 1 is: the slope of the relationship between Var (DG_i) and the midpoint of the diameter class ($\overline{DC}_i$) equals zero. Here again, a graph is helpful:

Null hypothesis 1:

H_o: $b = 0$

H_a: $b \neq 0$

(b is the slope of the variance regression line)

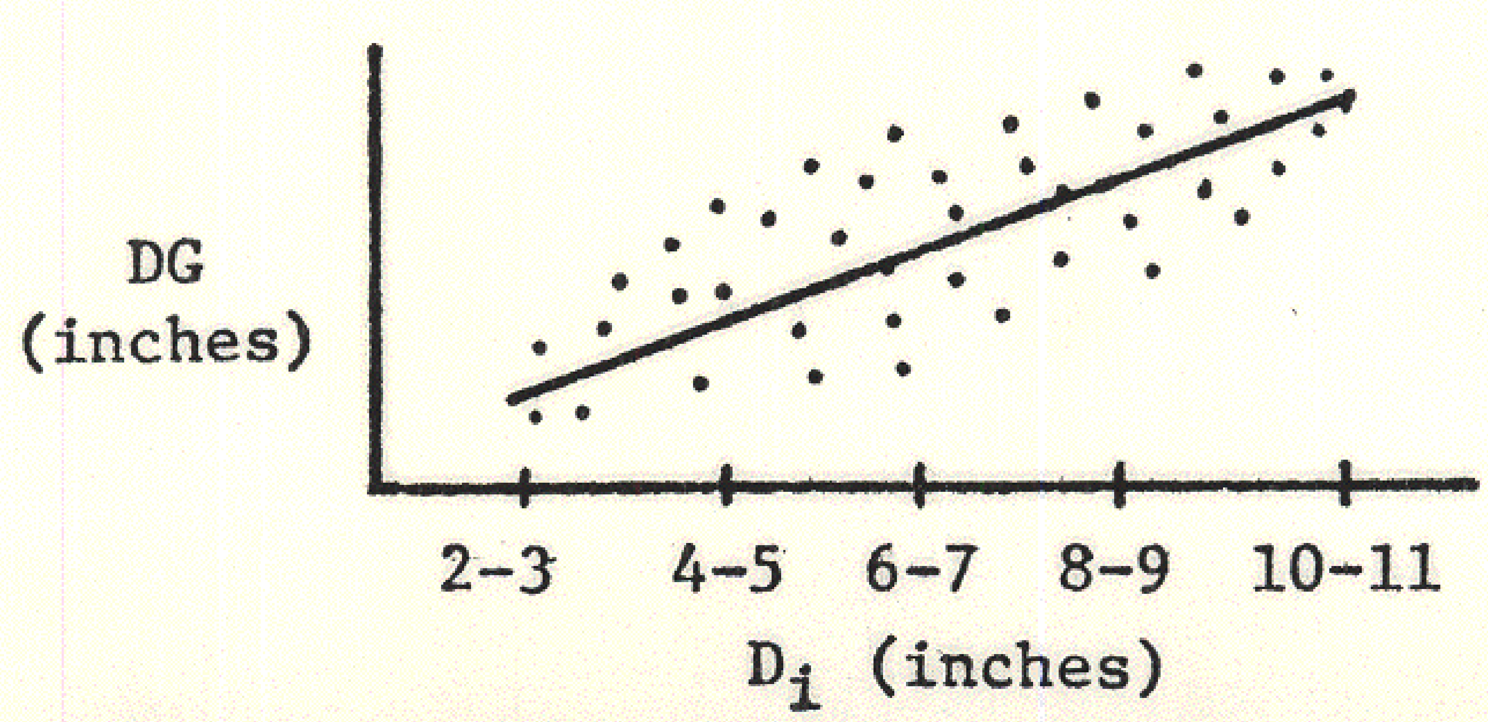


If null hypothesis 1 is rejected, it is useful to model the relationship between increment variance and diameter class. Such a model would provide estimates of Var (DG_i) for each diameter class. These estimates are then used to determine minimum sample size in stratified random sampling.

A second important assumption to test is that of <u>linearity</u> between diameter increment, DG, and initial tree diameter, D_1. If this assumption is valid, the following linear model is appropriate:

$$DG = a + bD_1.$$

For example, in the following diagram, a sample of trees drawn from only the 2-3 inch diameter class and the 10-11 diameter class may estimate b, the slope, as well as does a sample of trees drawn from all diameter classes.

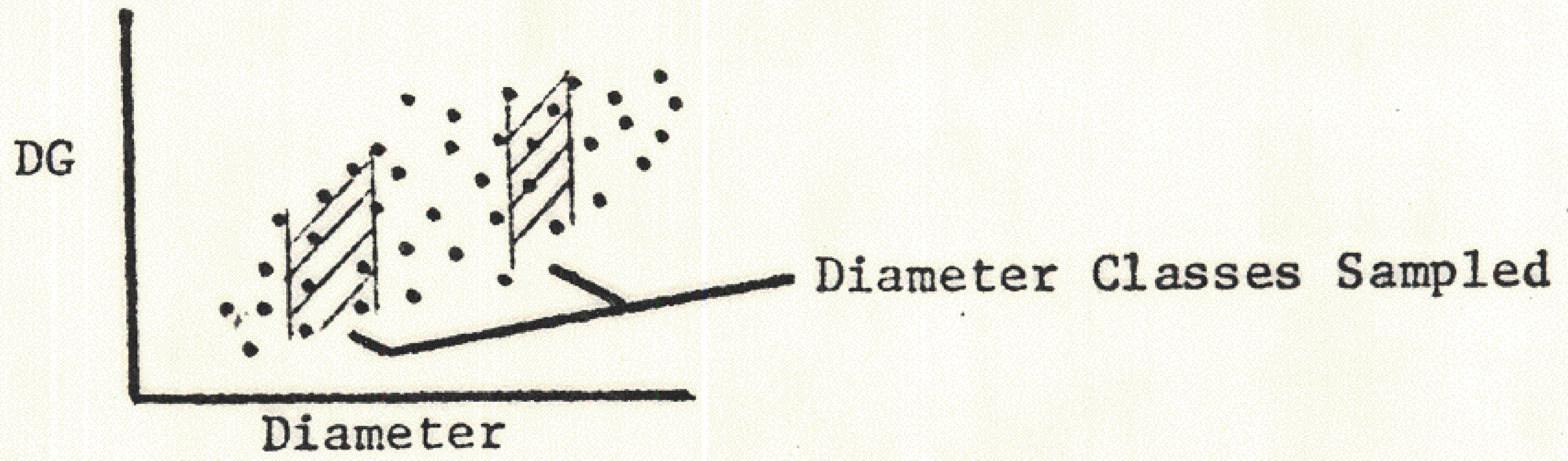


Null hypothesis 2 is that the relationship between DG and D_i is linear.

Phase II: Conduct Sampling Experiments to Test Several Alternate Sampling Plans

To conduct sampling experiments in the most efficient manner, computer sampling was used to repeatedly draw random samples. Turnbull (1968), among others, have performed similar Monte Carlo studies in which large numbers of samples were computer generated.

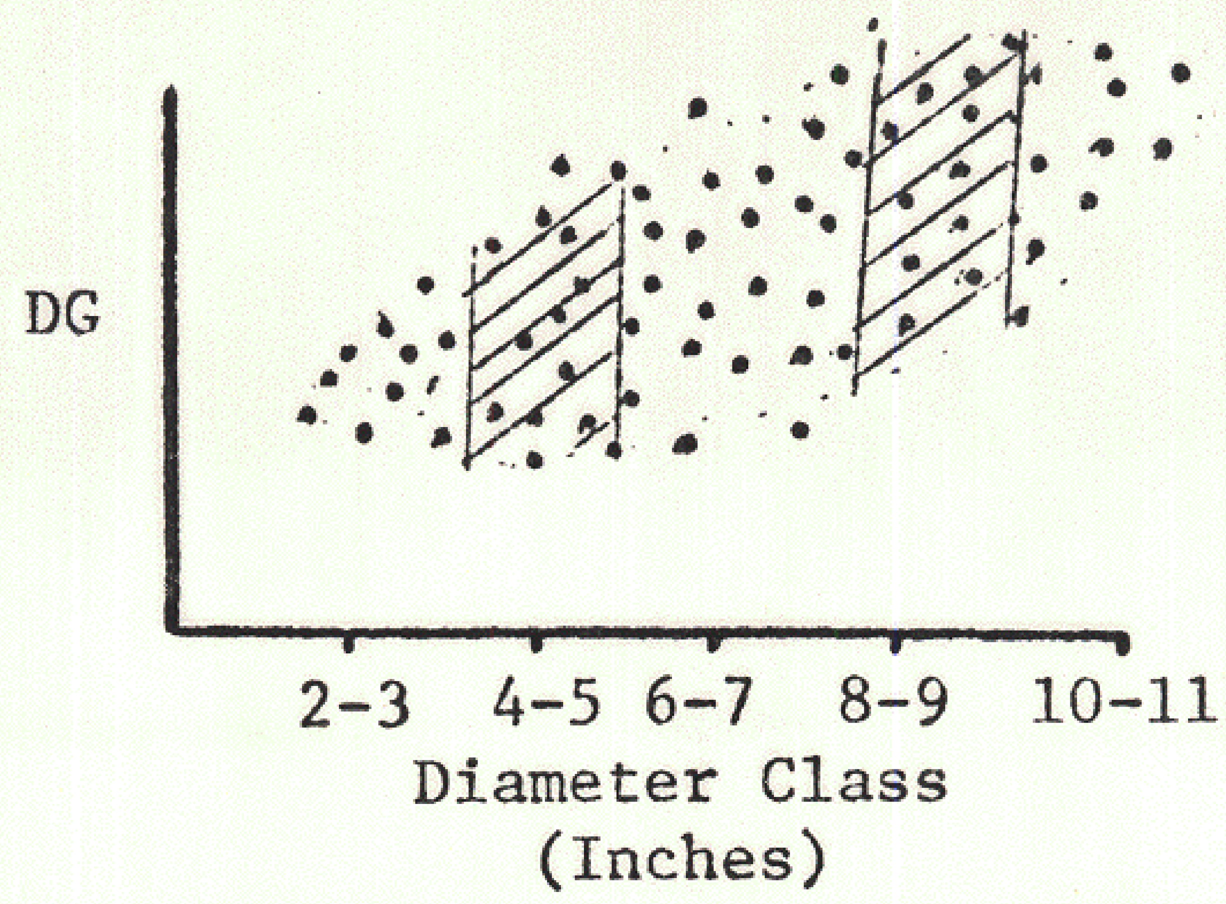
There is a critical sampling concept involved here which warrants explanation. The concept is to sample from diameter classes which yield representative estimates of Var (DG_i) and yet are towards the ends of the diameter distribution so as to reduce s_b^2. The following scattergram of diameter increment over diameter illustrates this sampling concept.



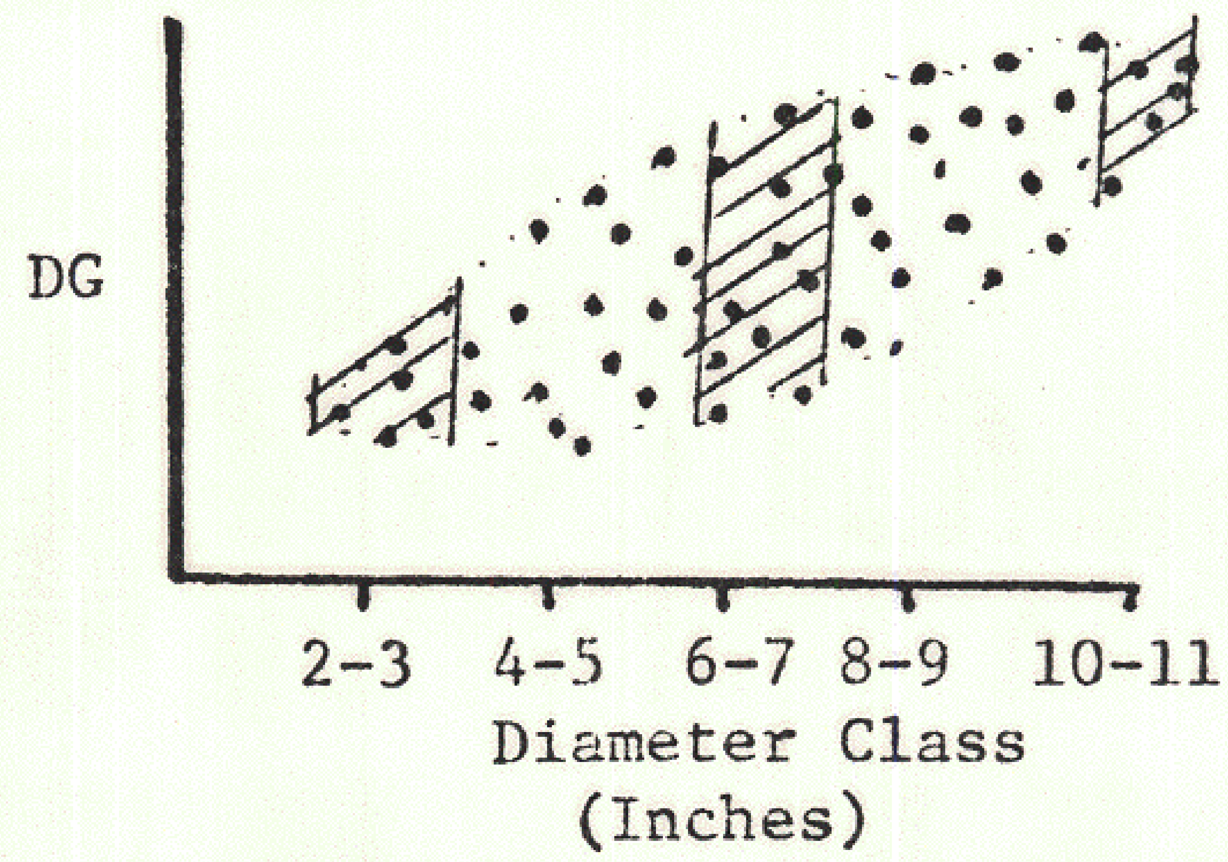
If the trend is linear (null hypothesis 2), it may suffice to sample from only those diameter classes on the ends of the diameter range.

A sample plan simply designates diameter classes from which to chose increment sample trees. Drawing from Hohenadl and Bartlett's experience, the following pattern of three sample plans was compared on each sample plot.

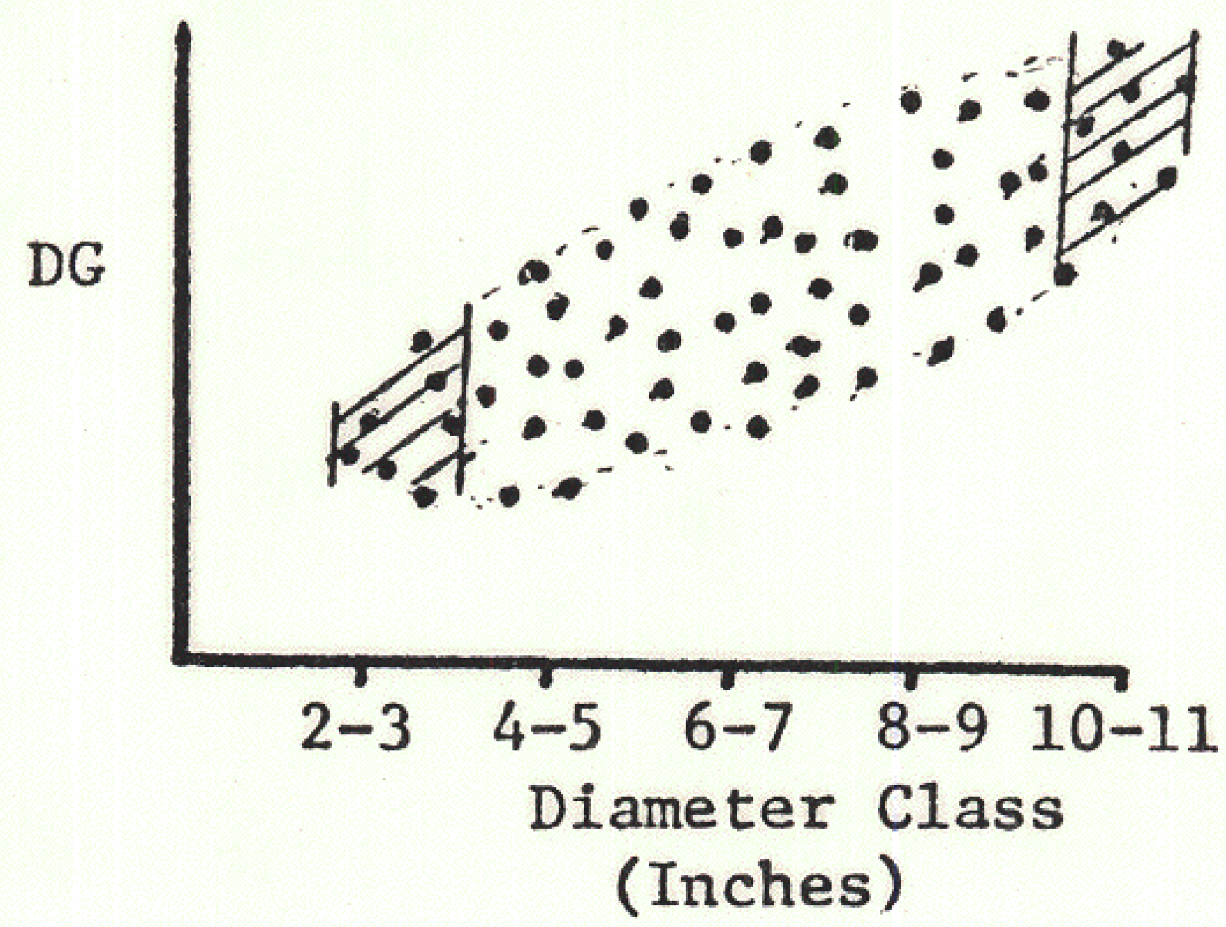
Sample Plan 1: Choose two disjoint diameter classes from the <u>middle</u> of the diameter range.



Sample Plan 2: Choose two diameter classes toward the <u>extremes</u> in the range plus one class at the <u>middle</u> of the diameter range.



Sample Plan 3: Choose only the two <u>extreme</u> diameter classes chosen in Sample Plan 2.



Experimental Procedure

To compare sample plans, the computer repeatedly sampled each plot 30 times for each sample plan. One sample consists of randomly choosing 5 trees from each diameter class specified by the respective sample plan. So sample plans 1 and 3 called for 10 trees per sample while sample plan 2 called for a 15-tree sample.

Next, the simple linear regression model:

$$DG = a + bD_1$$

was fitted to each of the 30 samples drawn for each sample plan. The computer stored the coefficients, a and b along with the residual mean square ($\hat{\sigma}^2_{DG}$) of each individual sample. This procedure was repeated for each sample plot.

Consequently, 30 regression estimates were derived, via computer sampling, for each sample plan on each sample plot. For each 30-sample set, the mean and standard error of the intercept (a), slope (b), and residual mean square ($\hat{\sigma}^2_{DG}$) were computed. These statistics were used to compare sample plans.

Criteria for Comparison of Alternate Sample Plans

Finally, each sample plan was tested by examining the relative merit of each plan. Relative merit was compared in the following terms:

1) Estimate of total basal area increment
2) Estimate of the slope, b, of the diameter increment trend
3) Estimate of diameter increment variance, $\hat{\sigma}^2_{DG}$
4) Estimate of the distribution of basal area increment

Test statistics used for terms (2) and (3) are the standard deviation of the slope, S_b, and the standard deviation of the estimated diameter increment variance, $S\hat{\sigma}^2_{DG}$, respectively. The most appropriate sample plan will be the one that gives the minimum S_b and $S_{\hat{\sigma}^2_{DG}}$.

Criteria (1), (2), and (4) refer to aspects of estimating the slope of the diameter increment trend. Criterion (3), in contrast, refers to the reliability of the estimate of variance of diameter increment.

It is important to have reliable estimates of the variance of diameter increment. Without these estimates, confidence intervals for the diameter increment trend cannot be constructed. Also, without such estimates, no tests for significant differences between increment trends can be made. Inability of sampling designs to provide reliable estimates of increment variance greatly restrict the design's usefulness when comparing treatment differences in terms of increment.

Four-year diameter increment data were obtained from permanent, one-tenth-acre control plots established as part of the University of Washington's Regional Forest Nutrition Project. Each field installation has a pair of control plots along with 4 treated plots.

Each sample plot, as defined for the present research, consists of diameter data from a pair of control plots. In other words, the effective area for each sample plot is .2 acre. Sixteen control plots were combined into eight sample plots.

Each control-plot-pair has essentially the same age and same site index. Maximum difference in 50-year site index was 4 feet. Maximum difference in breast height age was 2 years. Both site and age differences were within the standard error of the measured site and age.

The distribution of site and age classes represented by these eight sample plots is shown in Table 1, (Appendix). Breast height age ranged from 23 to 50 years. 50-year site index varied from 105 to 150 feet. One sample plot (.2 acre) represents each site-age combination indicated. The number in parenthesis ( ) is the sample plot serial number.

The number of trees sampled in each sample plot is also displayed by age class and site class in Table 2, (Appendix). The diameter increment data studied here was obtained by difference between periodic diameter tape measurements from 1072 trees ranging in diameter from 1.6 inches to 17.0 inches. The number of trees in each 2-inch diameter class for each plot is shown in Table 3, (Appendix).

SCOPE OF THE INVESTIGATION

This study is limited to investigating the trend of average diameter increment by diameter class ($\overline{DG}_i$). Conventional fixed-area plot methods are assumed to adequately estimate distribution of stems by diameter class (N_i).

The distribution of tree mortality and ingrowth has been excluded from this study. Only increment from survivor trees has been examined.

A brief discussion about populations and sampling follows. Cochran (1963) defines a population as the aggregate from which a sample is chosen. It is important that the population to be sampled (sample population) coincide with the population from which information is required (target population). In this study, the bivariate population selected as target comprises the diameters and annual diameter increments of all trees in natural, untreated, even-aged, single-site stands of Douglas-fir in Washington and Oregon. The bivariate sample population comprises the diameters and periodic annual diameter increments of all trees on natural, untreated, even-aged, single-site, permanent sample plots on which at least 80% of the basal area is Douglas-fir. Periodic annual diameter increment was computed from single 4-year growth periods. For this reason, results are not assumed to apply to consecutive remeasurements beyond a 4-year period.

The assumptions of uniform variance (homoscedasticity) and linearity are valid on six of the eight sample plots. Further, sample plan 2 is the most appropriate sampling design to follow. These results are discussed in detail below:

Distribution of Variance About the Diameter Class Mean Increment, Var (DG_i)

Figure 1, in the appendix, displays the relationship of Var (DG_i) to diameter class for each sample plot. As one might suspect, the trend is slightly upward as tree size increases. Tests of null hypothesis 1 (no correlation between Var (DG_i) and diameter, page 10) for each sample plot gave the following results:

Sample Plot	Slope of Variance Regression Line	Correlation Coefficient	t-statistic
1	.000136	.4638	.91
2	.000487	.8712	3.55*
3	.000508	.6800	2.07
4	.000353	.8250	3.26*
5	.000135	.3877	.73
6	.000232	.4928	1.13
7	.000194	.7823	2.51
8	.000206	.6905	2.34

* Significant correlation at .05 level

This test procedure is presented in Draper and Smith (1966). The t-statistic on sample plots 2 and 4 was just inside the critical region for rejecting the null hypothesis. This is by no means a conclusive test for homoscedasticity. Although, on a plot basis, the data indicate 75% of the time the assumption of uniform variance of diameter increment is valid.

Test of the Linear Relationship Between Diameter Increment, DG, and Initial Diameter, D_1.

The test for linearity as outlined in Snedecor and Cochran (1967) gave the following F-statistics:

Sample Plot	F-statistic
1	3.6
2	4.0
3	1.8
4	3.1
5	1.3
6	4.7
7	12.2*
8	9.5*

* significant deviation from linearity at .05 level

Only the two sample plots in age class 41-50 deviated from linearity significantly. This deviation was caused, in part, by an abnormal number of trees less than 7.5 inches in diameter which showed no diameter growth over the 4-year period. On sample plot 7, 64 of 153 (42%) of these size trees did not grow in diameter. On sample plot 8, 29 of 80 (36%) of these trees did not grow in diameter. This high occurence of zero growth lowered the average diameter class increment for these smaller classes which, in turn, influenced the shape of the overall trend.

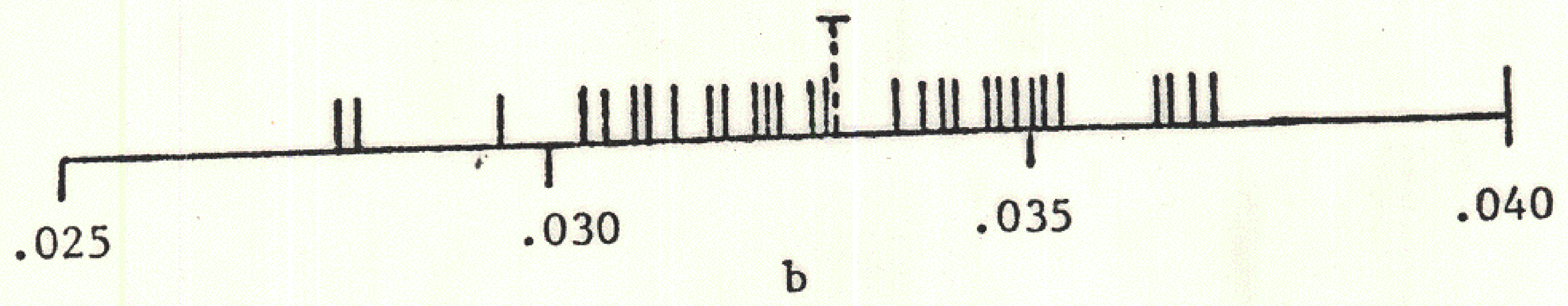
On all other sample plots the trend of average diameter increment, $\overline{DG}_i$, is linear with respect to diameter class. Here again, on a plot basis, 75% of the time the assumption of linearity is valid. Figure 2 in the Appendix shows the relationship of $\overline{DG}_i$ to diameter class for each sample plot grouped by age class.

Comparison of Alternate Sample Plans

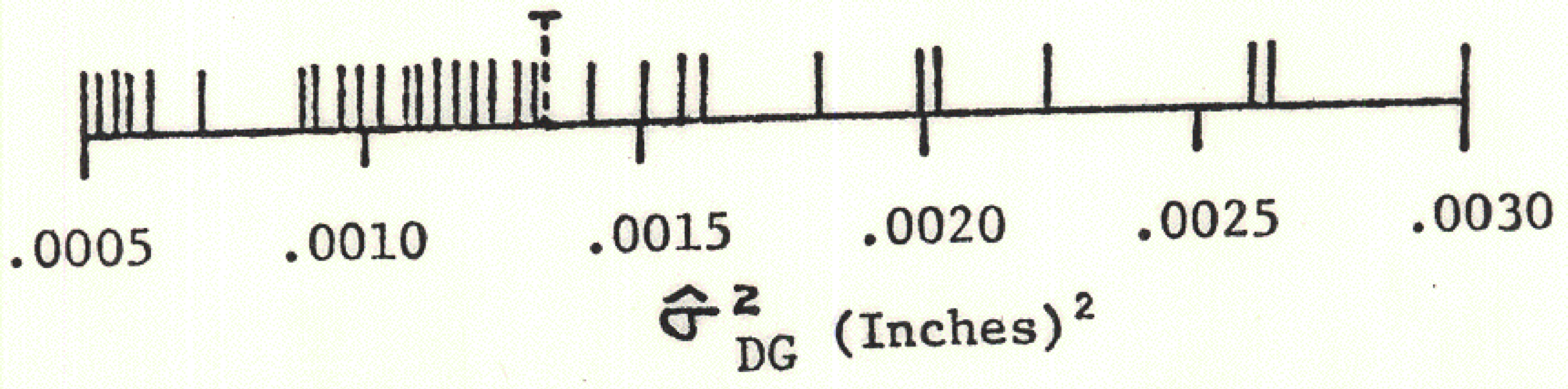
All sample plans obtained unbiased estimates of slope, β and unbiased estimates of variance of diameter increment, σ^2_{DG}. These estimates were closely grouped about their means.

Sample Plan 2 generally gave the closest grouping. Distribution of the 30 sample estimates of β and σ^2_{DG} is illustrated below for sample plan 2 when applied to sample plot 1. The means of the estimate distributions are indicated by a dashed vertical line.

Distribution of Slope Estimates



Distribution of Estimates of Diameter Increment Variance



These 30-sample distributions were obtained for all three sample plans on each of the eight sample plots. From these distributions, means and standard deviations were calculated.

Criterion 1: Estimate of Total Basal Area Increment

Sample Plan 2 came closest to estimating total basal area increment on four of the eight sample plots. (See Figure 3 in the Appendix). Each estimate of total basal area increment was, on the average, within 11% of the whole plot total basal area increment. The breakdown by sample plan is shown below.

Sample Plan	Average of Percent Differences
1	6.6
2	6.8
3	10.9

Note that, on the average, sample plans 1 and 2 essentially estimate total basal area increment equally well.

Criterion 2: Estimate of the Slope, b, of the Diameter Increment Trend

Sample plans 2 and 3 estimated slope of the diameter increment trend equally well on all sample plots. In contrast, sample plan 1 generally overestimated slope. These estimates were compared to the slope estimate derived from measuring all trees on the sample plot (whole plot estimate of slope). Figure 4 in the Appendix shows these comparisons.

Note also the comparison of the standard deviation of the slope estimate, S_b. The largest disturbance in the slope estimate, expressed in terms of S_b, was experienced from sample plan 1. The less the disturbance, the more reliable the estimate. On this basis, sample plan 2 provides the most reliable estimate of slope.

As a matter of interest, the standard deviation of the slope as computed from Bartlett's formula:

$$S_b = \sqrt{\frac{2\hat{\sigma}^2}{K(\bar{X}_3 - \bar{X}_1)^2}}$$

was compared to the standard deviation of the slope derived empirically. Results are presented in Table 4 in the Appendix. Bartlett's S_b compared favorably with the empirical S_b. This is one of the few comparisons of Bartlett's formula against actual data. The results of this comparison lend credibility to his work.

Criterion 3: Estimate of Variance of Diameter Increment, σ^2_{DG}

Sample plan 2 came closest to estimating variance of diameter increment, σ^2_{DG}, on most sample plots. It also produced estimates with least disturbance in terms of the standard deviation of the variance of diameter increment, $S\hat{\sigma}^2_{DG}$ (see Figure 5). Here again, on the basis of $S\hat{\sigma}^2_{DG}$, sample plan 2 provides the most reliable estimate of variance of diameter increment.

Criterion 4: Estimate of Distribution of Total Basal Area Increment

The distribution of total basal area increment was estimated equally well from both sample plan 2 and sample plan 3. Estimated distributions are compared to whole plot distributions for each sample plot in Figures 6 through 13 in the Appendix. Sample plan 1 generally underestimated basal area increment on smaller trees and overestimated basal area increment on larger trees, an expected consequence of overestimates of the slope of diameter increment relative to diameter.

Sample plans 2 and 3 provided similar estimates of BG_i, as expected, since estimates of b from these two sample plans were nearly identical.

SUMMARY AND CONCLUSIONS

Based on the four criteria for comparison of sample plans, the relative merit of each plan is displayed in terms of the following symbols:

+ indicates relatively good plan to use

- indicates relatively poor plan to use

Criterion	Sample Plan 1	2	3
1) Estimate of total basal area increment	+	+	-
2) Estimate of the slope, b, of the diameter increment trend	-	+	+
3) Estimate of variance of diameter increment	-	+	-
4) Estimate of the distribution of basal area increment	-	+	+

Note that only sample plan 2 was consistently a good plan to use. It was the plan which provided the most stable estimate of variance of diameter increment, in terms of standard deviation of that estimate (refer to Figure 5 in the Appendix).

It is important to remember that in order to construct confidence intervals for diameter increment estimates and to test differences between diameter increment trends, a reliable (stable) estimate of the variance of diameter increment is necessary. On this basis, sample plan 2 is the best overall plan to use. Sample plan 2 calls for randomly choosing 5 trees from each of three 2-inch diameter classes, one class from each extreme of the diameter range plus one class in the middle of the diameter range.

This sample plan provides estimates of total basal area increment per acre per year with 7% of the whole plot total basal area increment per acre per year.

Assumptions of homoscedasticity and linearity need further inspection. On six of the eight sample plots these assumptions were valid. This is encouraging but not conclusive. However, the t-statistic used to test homoscedasticity was just inside the critical region for rejection on sample plots 2 and 4. Since this estimation procedure relies heavily on the test of linearity, further testing of this relationship from more data is recommended.

There is a potential savings in time and cost in remeasurement of permanent sample plots by using sample plan 2. Reliable, stable estimates of (a) diameter increment trend, (b) total basal area increment, (c) total basal area increment distribution, and (d) variance of diameter increment are obtained by remeasuring less than a <u>third</u> of the trees (15 sample trees) on conventional fixed-area plots.

There is a range in the required reliability of growth estimates associated with the purpose of obtaining these estimates. Required reliability of individual plot growth estimates is perhaps the least in the continuous forest inventory (CFI) procedures for estimating differences between growing stock totals on two successive occassions of measurement. On the other hand, required reliability may be at its highest where individual tree increment is required for every tree, as in experimental thinning based on individual tree increment.

CFI — All trees tagged

low reliability — high reliability

The sampling plans examined here provide reliability somewhat in the middle of this range. Estimates of diameter increment relative to diameter provide growth comparisons related to stand structure in the form of basal area increment distribution. This allows comparisons in relation to tree size (grouped by diameter classes) as well as in relation to total per acre differences.

When comparing increment trends, previous diameter increment is often used as a covariate. Increment cores are taken from sample trees to establish these previous increment trends. Application of sample plan 2 to select these increment core trees can save considerable time and cost without undue sacrifice in precision of past growth estimates. This results from boring only 15 trees on the sample plot. Depending on stand density, this usually amounts to only boring one-tenth to one-third of the trees on the plot. In addition, this sample plan yields stable estimates of variance of diameter increment required for tests of significant differences between diameter increment trends.

Another possible application is to combine partial remeasurement of the plot (sample trees only) with periodic remeasurement of all trees on the plot. This procedure provides a means to re-establish total information on the stand structure regularly over several years. But the necessary expense of re-measurement for interim information is reduced by partial remeasurement.

To use this plan from the establishment of the plot, the forester must remember ingrowth and mortality must be sampled independently from survivor increment. The following procedure is suggested. Arbitrary threshold of ingrowth is 1.6 inches DBH.

Initial Measurement of Sample Plot

1) Measure all live trees 1.6 inches DBH or larger to establish a 2-inch diameter class tally.
2) Tag all trees between 1.6 and 2.0 inches DBH.
3) Blaze all dead trees.
4) Randomly select and tag 5 sample trees from the smallest diameter size class; 5 sample trees from the middle diameter size class; and 5 sample trees from the largest diameter size class.

Subsequent Remeasurement of the Sample Plot

1) Remeasure diameters of all sample trees.
2) Scan all small trees and record as ingrowth any untagged trees between 1.6 and 2.0 inches DBH.
3) Any dead trees not blazed are counted as mortality. Blaze all dead trees. Dead sample trees should be replaced by selecting any live

tree which was in that same 2-inch diameter class at the time of initial measurement. This is determined by increment boring.

The entire initial diameter tally is then updated by using the trend line estimated from sample tree diameter increment data. This is similar to conventional CFI updating methods when only a fraction of the entire inventory is remeasured.

A word of caution is in order. When applying sample plan 2 to treated stands, we assume the same intensity of measurements (15 trees) will provide unbiased estimates of the variance of diameter increment. This assumption is probably valid for stand treatments that do not include thinning. It may not be valid for thinning treatments, however, since thinning removes a segment of the stem distribution. Additional study is needed to quantify the effect of stand treatment on the distribution of the variance of diameter increment.

RECOMMENDATION FOR FURTHER STUDY

The methodology developed in this study may be used to further investigate and refine procedures used to estimate forest growth. The approach is not restricted by species, growth parameter, or stand condition.

One logical extension is to examine sampling designs used to estimate total volume increment and total volume increment distribution within the stand. Variance of volume increment would be investigated to determine the appropriate sampling scheme. Heights as well as diameters of sample trees would be measured.

Another relationship to establish is effect of various stand treatments on the distribution of variance of diameter increment. The null hypothesis is that stand treatment does not alter the distribution of this variance.

Comparing additional sampling designs with those tested in this investigation is also a theme for further study. One such design might consist of randomly choosing 10 sample trees across the entire range of diameter classes. Another plan might call for sampling only three trees from each diameter class represented on the plot. When making these comparisons, it is important to remember the assets of reliable sampling plans as outlined in this study. These qualities are unbiased, stable estimates of the following:

1) Total basal area increment
2) Total basal area increment distribution
3) Slope of the diameter increment trend
4) Variance of diameter increment

Finally, the assumptions of linearity and uniform variance associated with the estimation procedure need further investigation. Since this procedure relies heavily on the test of linearity, further testing of this relationship from more data is recommended.

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TABLE 1

DISTRIBUTION OF THE 8 SAMPLE PLOTS

BY SITE AND AGE CLASS

(Plot serial number in parenthesis)

Breast Height Age Class (years)	50-Year Site Index (feet) 105-115	116-134	135-150
23-30	(1)	(2)	(3)
31-40	(4)	(5)	(6)
41-50	(7)	(8)	-

TABLE 2

DISTRIBUTION OF NUMBER OF TREES IN SAMPLE PLOTS

BY SITE AND AGE CLASS

Breast Height Age Class (years)	50-Year Site Index (feet) 105-115	116-134	135-150	Total
23-30	146	128	127	401
31-40	122	92	92	306
41-50	189	176	-	365
Total	457	396	219	1072

TABLE 3

DIAMETER TALLY BY 2-INCH

DIAMETER CLASSES BY SAMPLE PLOT

Sample Plot	Diameter Class (inches)	No. of Trees
1	2-3	24
	4-5	52
	6-7	41
	8-9	19
	10-11	10
Total		146
2	2-3	16
	4-5	33
	6-7	30
	8-9	29
	10-11	14
	12-13	6
Total		128
3	2-3	6
	4-5	10
	6-7	24
	8-9	39
	10-11	20
	12-13	18
	14-15	10
Total		127
4	2-3	5
	4-5	10
	6-7	29
	8-9	27
	10-11	22
	12-13	19
	14-15	10
Total		122

Sample Plot	Diameter Class (inches)	No. of Trees
5	6-7	7
	8-9	16
	10-11	26
	12-13	26
	14-15	17
Total		92
6	6-7	12
	8-9	20
	10-11	16
	12-13	17
	14-15	13
	16-17	14
Total		92
7	2-3	67
	4-5	52
	6-7	34
	8-9	12
	10-11	14
	12-13	10
Total		189
8	2-3	33
	4-5	20
	6-7	27
	8-9	28
	10-11	24
	12-13	25
	14-15	10
	16-17	9
Total		176

TABLE 4

Standard Deviation of the Slope, S_b, of Diameter Increment Trend Computed from Bartlett's Formula Compared to S_b Derived Empirically from Each Sample Plan by Sample Plot.

Sample Plot	Sample Plan	Sample S_b	Bartlett's S_b
1	1	.00576	.00682
	2	.00244	.00466
	3	.00245	.00412
2	1	.00857	.00752
	2	.00119	.00650
	3	.00110	.00676
3	1	.00542	.00638
	2	.00207	.00302
	3	.00200	.00288
4	1	.00802	.00757
	2	.00283	.00352
	3	.00301	.00384
5	1	.00369	.00440
	2	.00279	.00363
	3	.00272	.00309
6	1	.00851	.00882
	2	.00322	.00355
	3	.00321	.00285
7	1	.00464	.00590
	2	.00198	.00259
	3	.00211	.00212
8	1	.00466	.00489
	2	.00133	.00223
	3	.00129	.00169

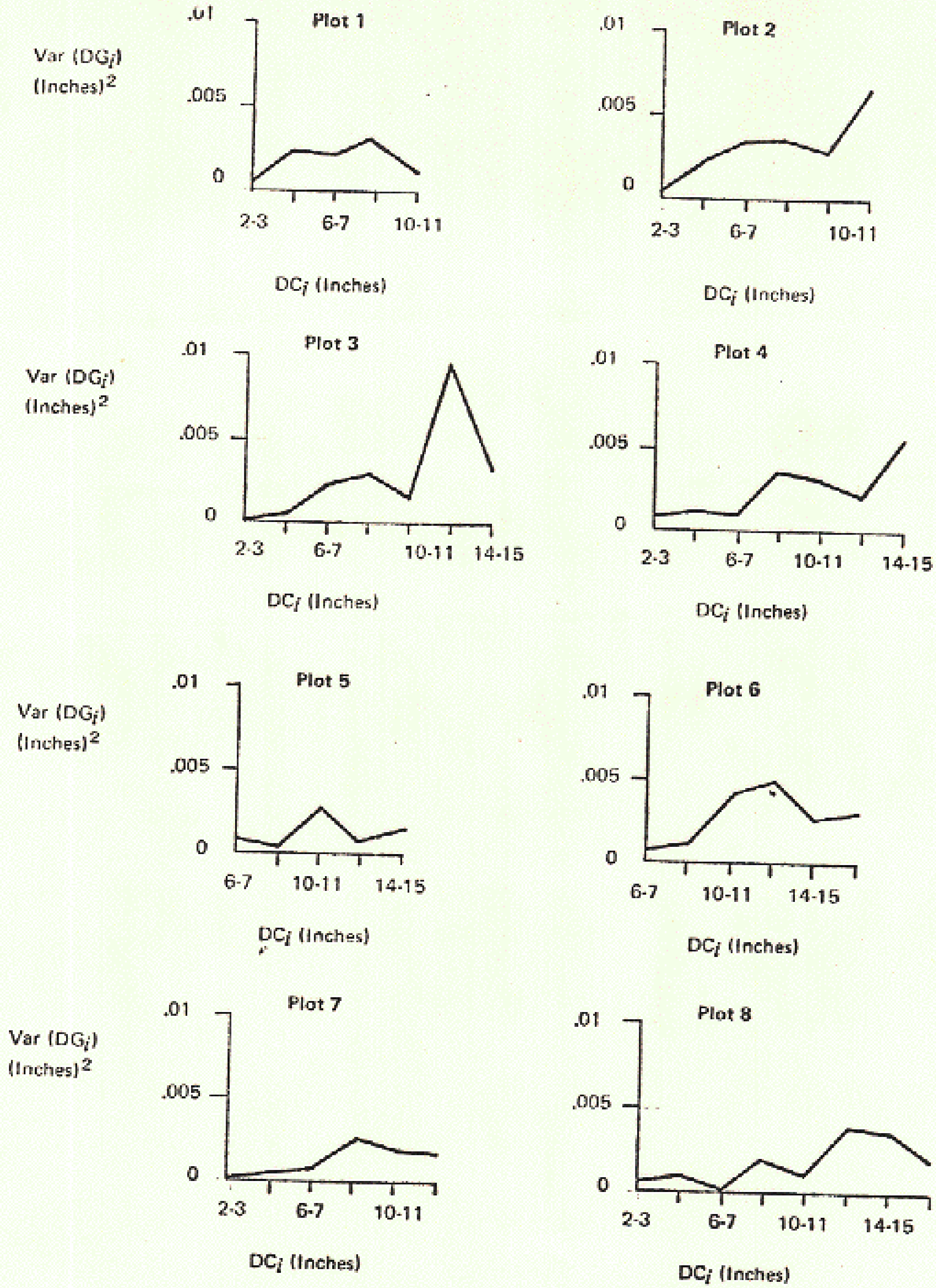

Figure 1. Variance of diameter increment [Var (DG$_i$)] by diameter class [DC$_i$] for each sample plot.

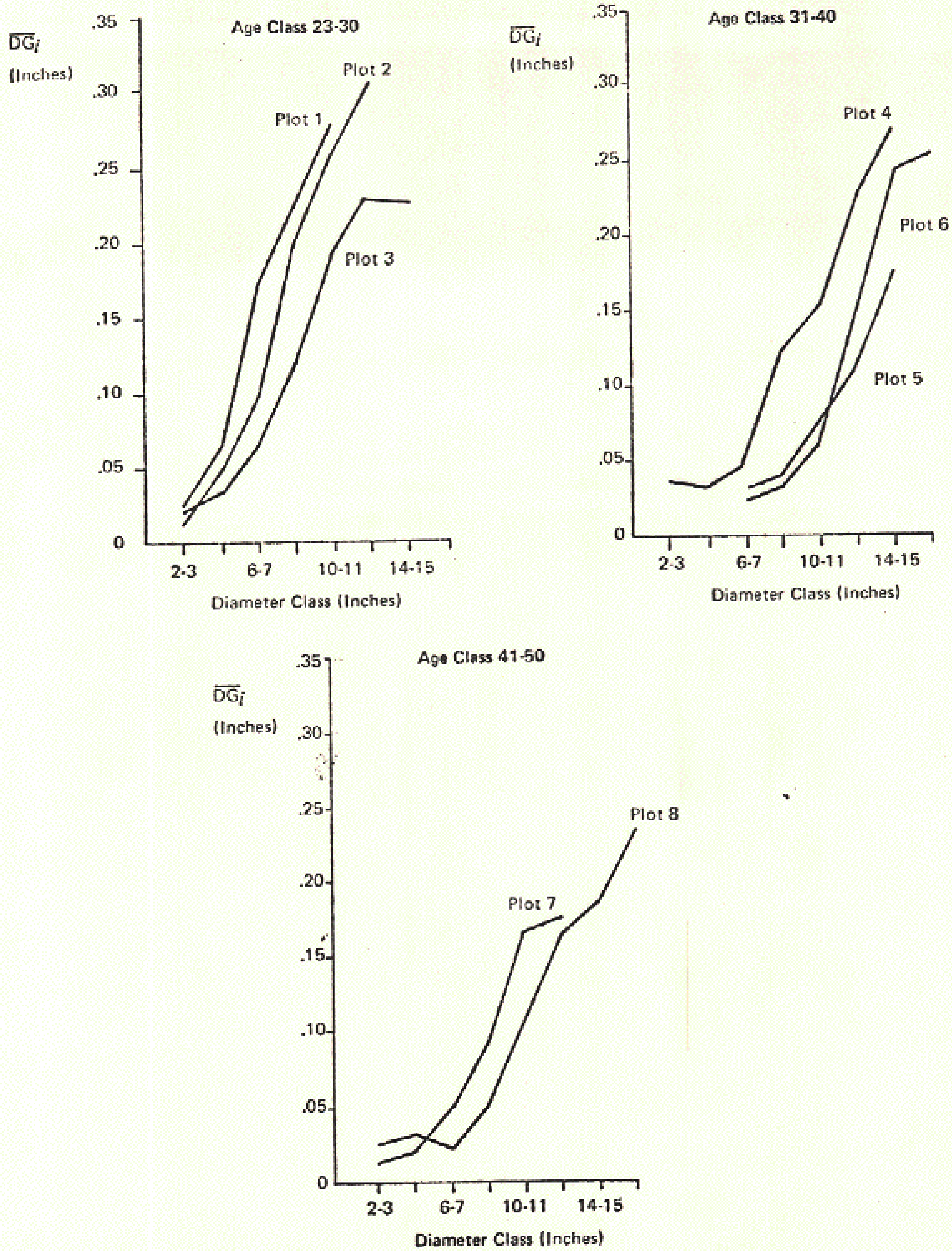

Figure 2. Average annual diameter increment [$\overline{DG}_i$] relative to diameter class by sample plot and age class.

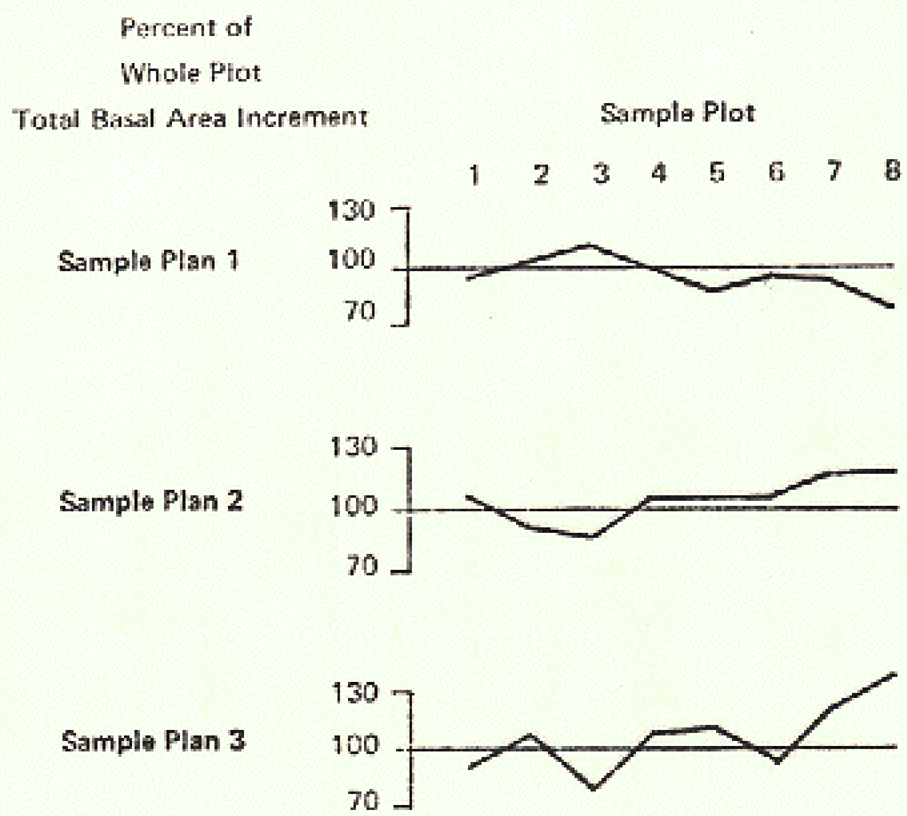

Figure 3. Comparison of estimates of total basal area increment per acre per year by sample plot as a percent of the whole plot total basal area increment per acre per year.

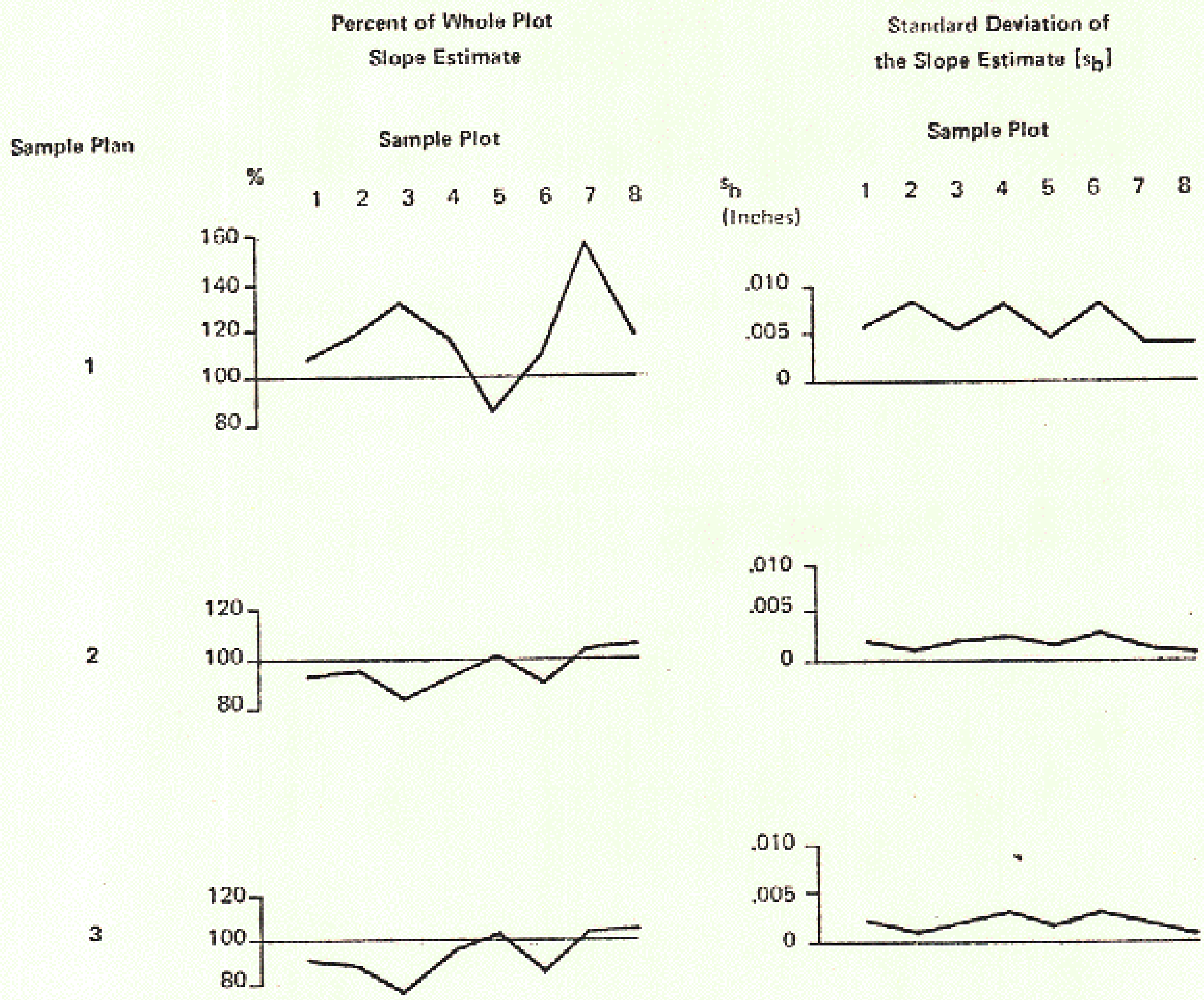

Figure 4. Comparison of estimates of slope [b] of the diameter increment trend relative to diameter.

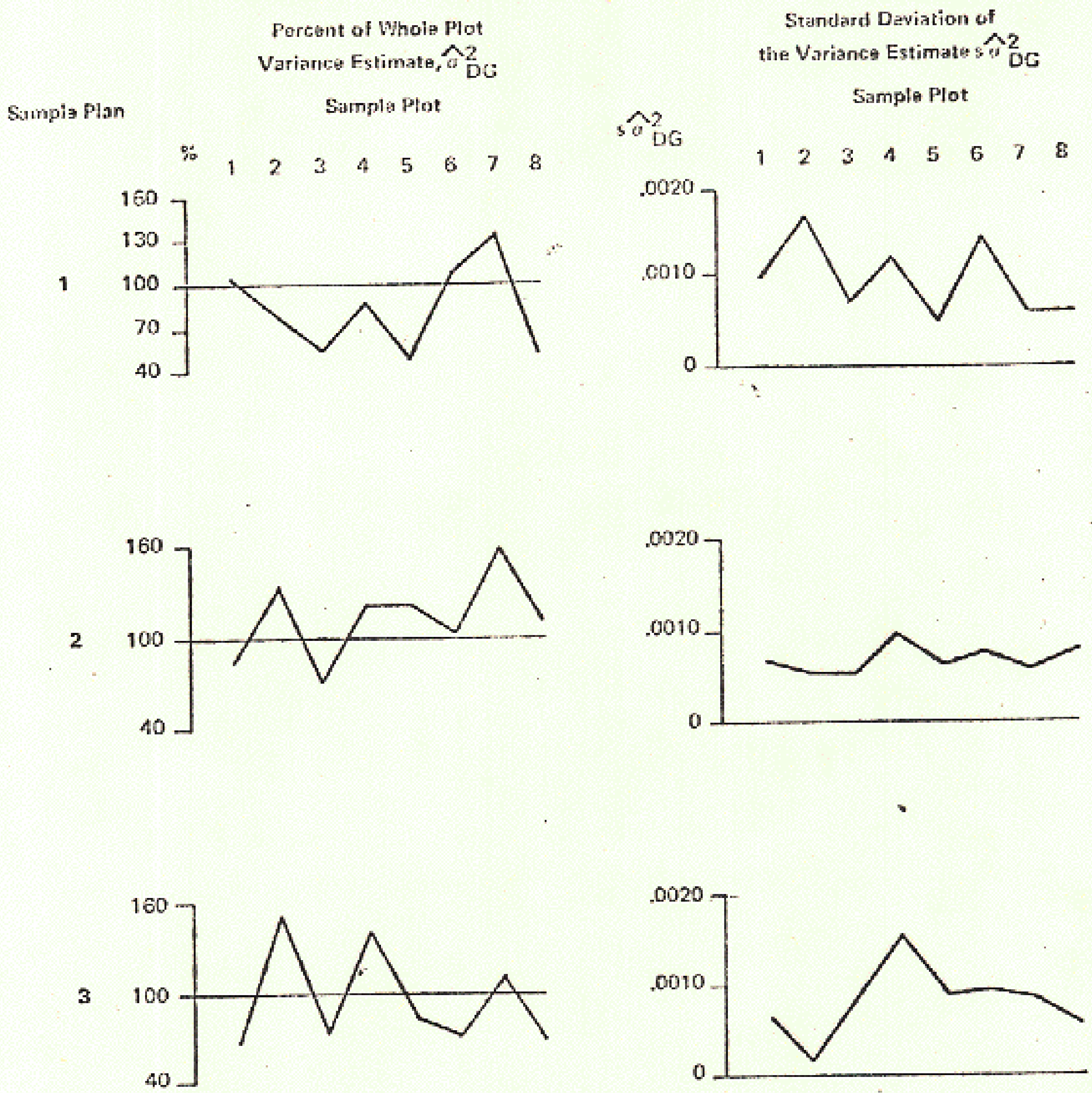

Figure 5. Comparison of estimates of diameter increment variance [$\hat{\sigma}^2_{DG}$].

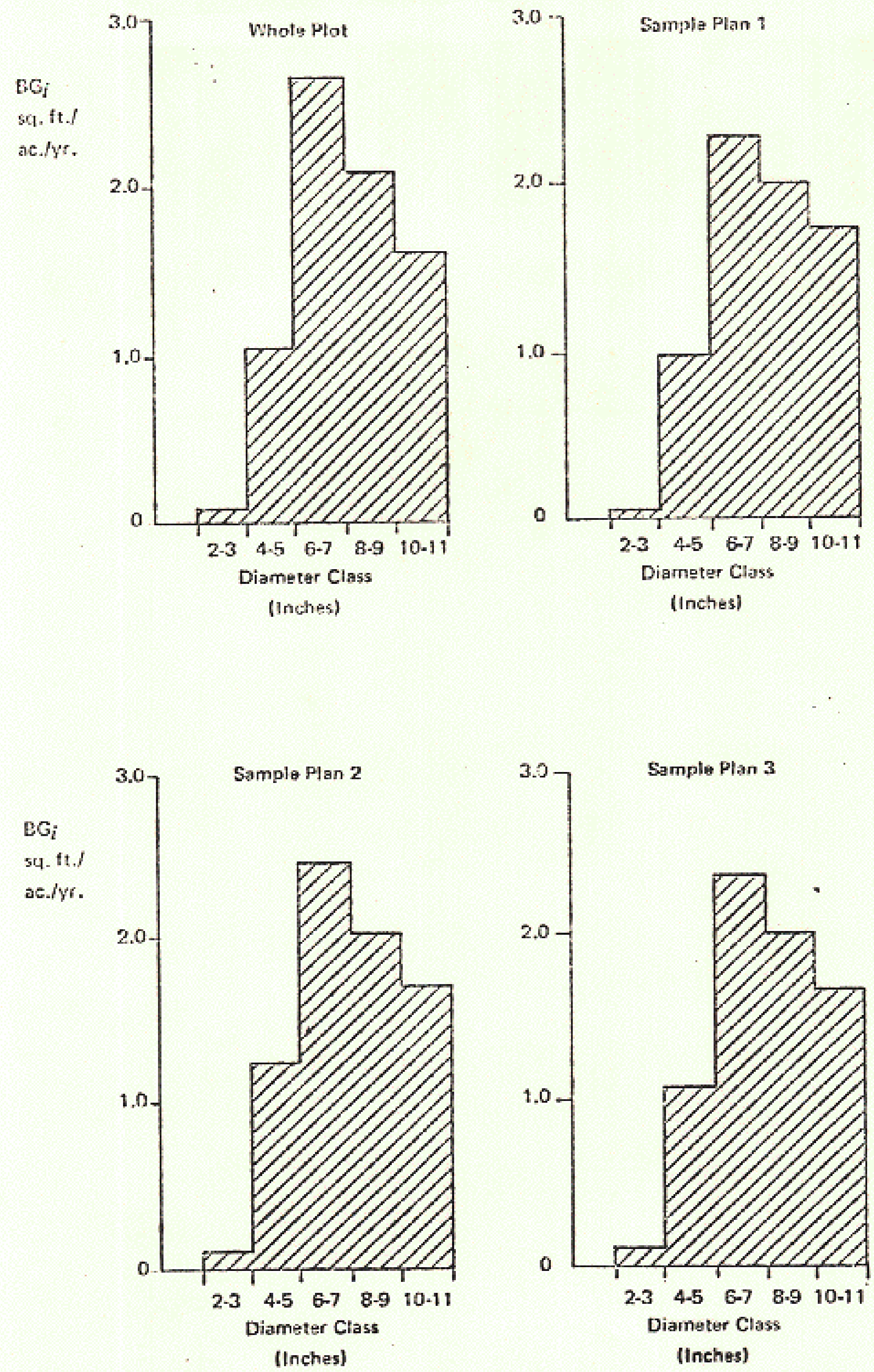

Figure 6. Comparison of estimates of basal area increment distribution $[BG_i]$ per acre per year for sample plot 1.

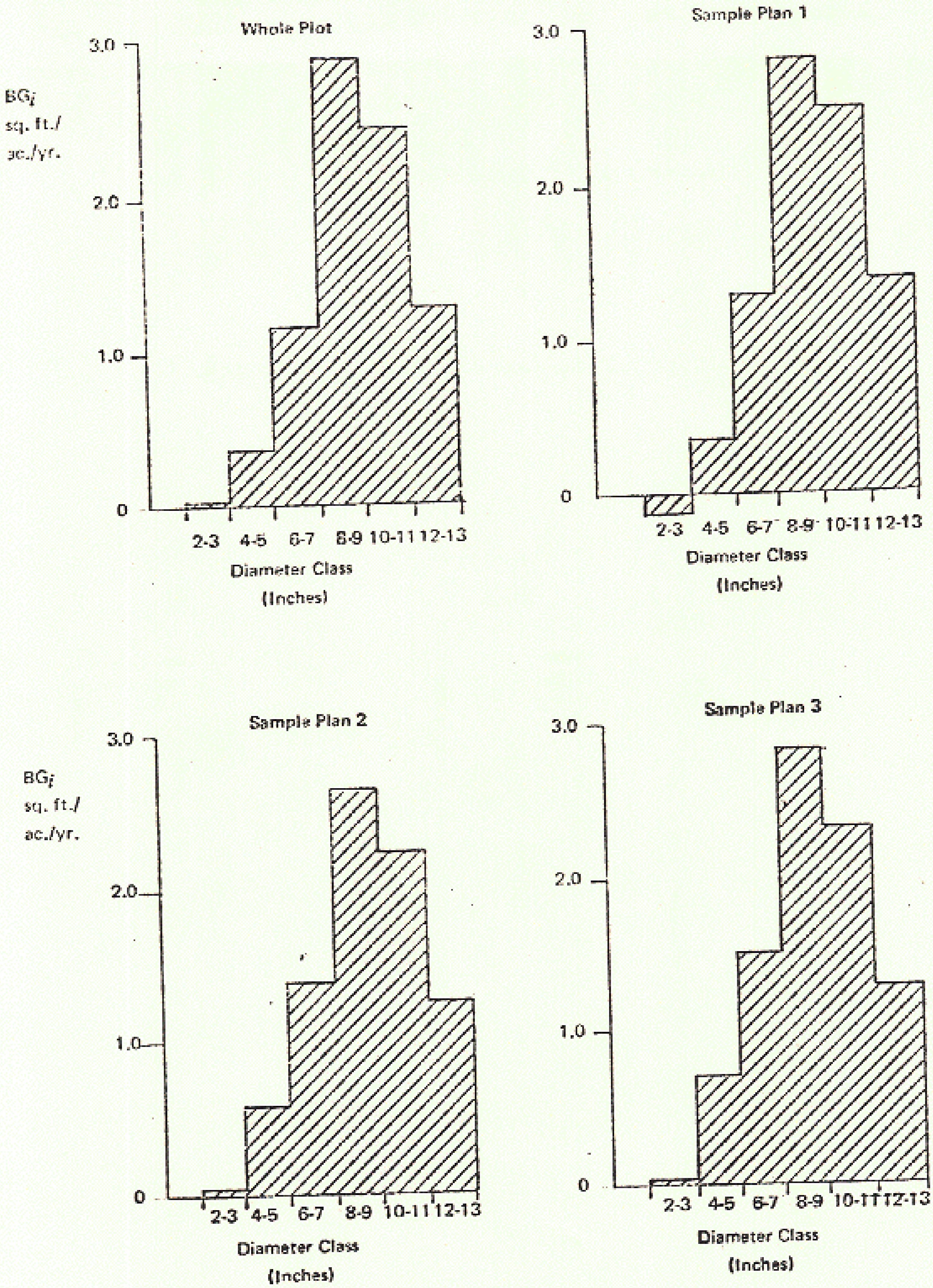

Figure 7. Comparison of estimates of basal area increment distribution (BG$_i$) per acre per year for sample plot 2.

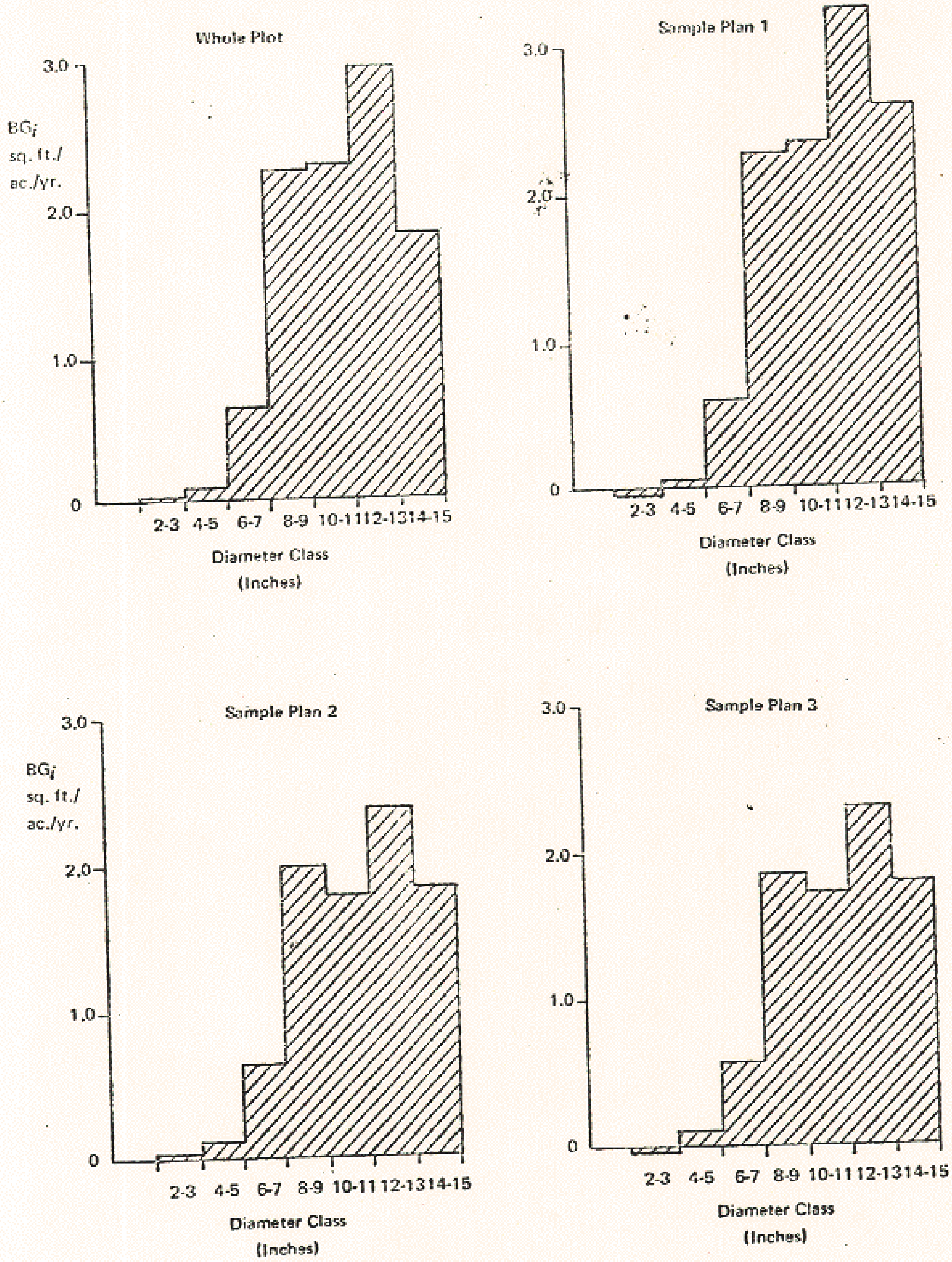

Figure 8. Comparison of estimates of basal area increment distribution, [BG$_i$] per acre per year for sample plot 3.

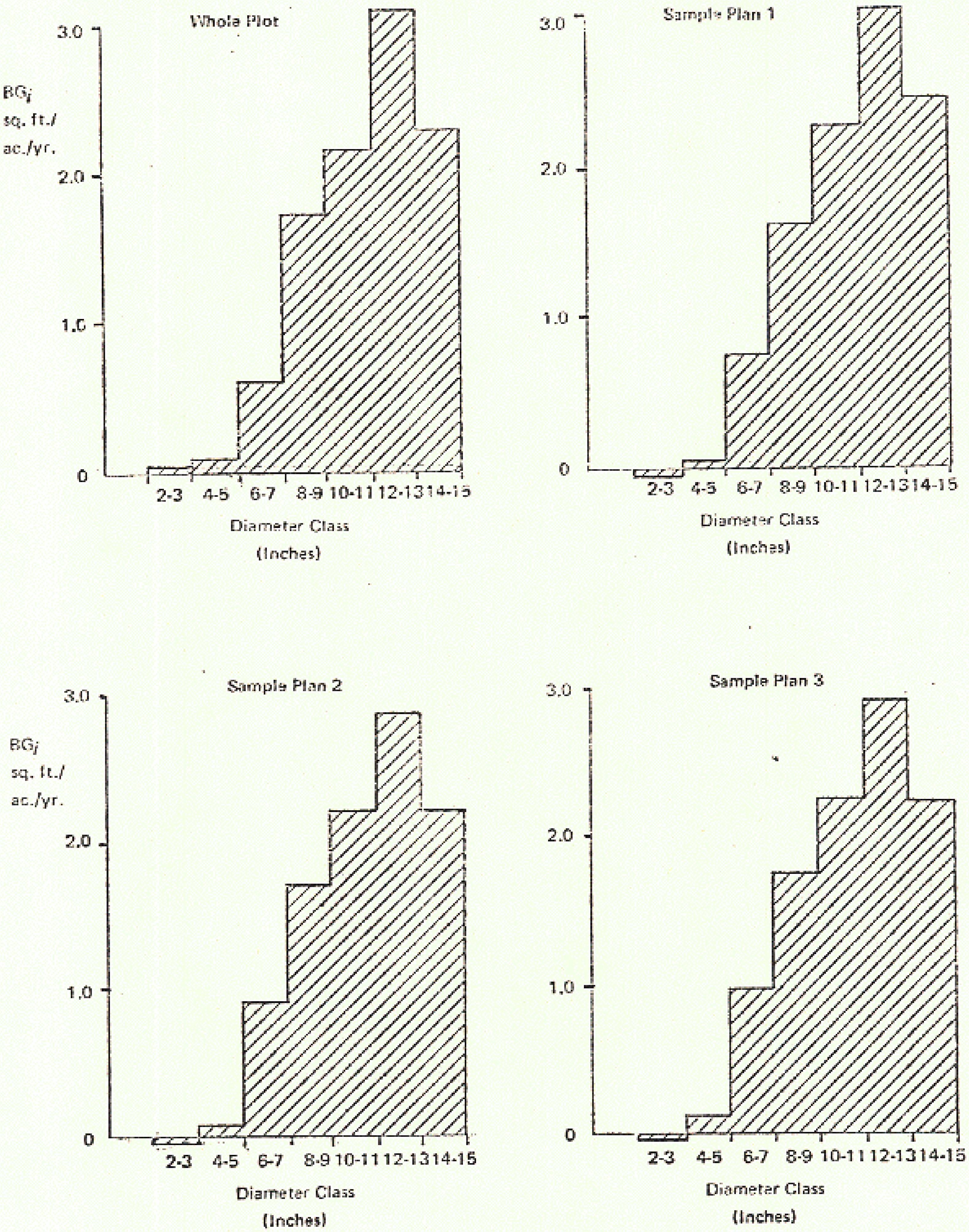

Figure 9. Comparison of estimates of basal area increment distribution (BG_j) per acre per year for sample plot 4.

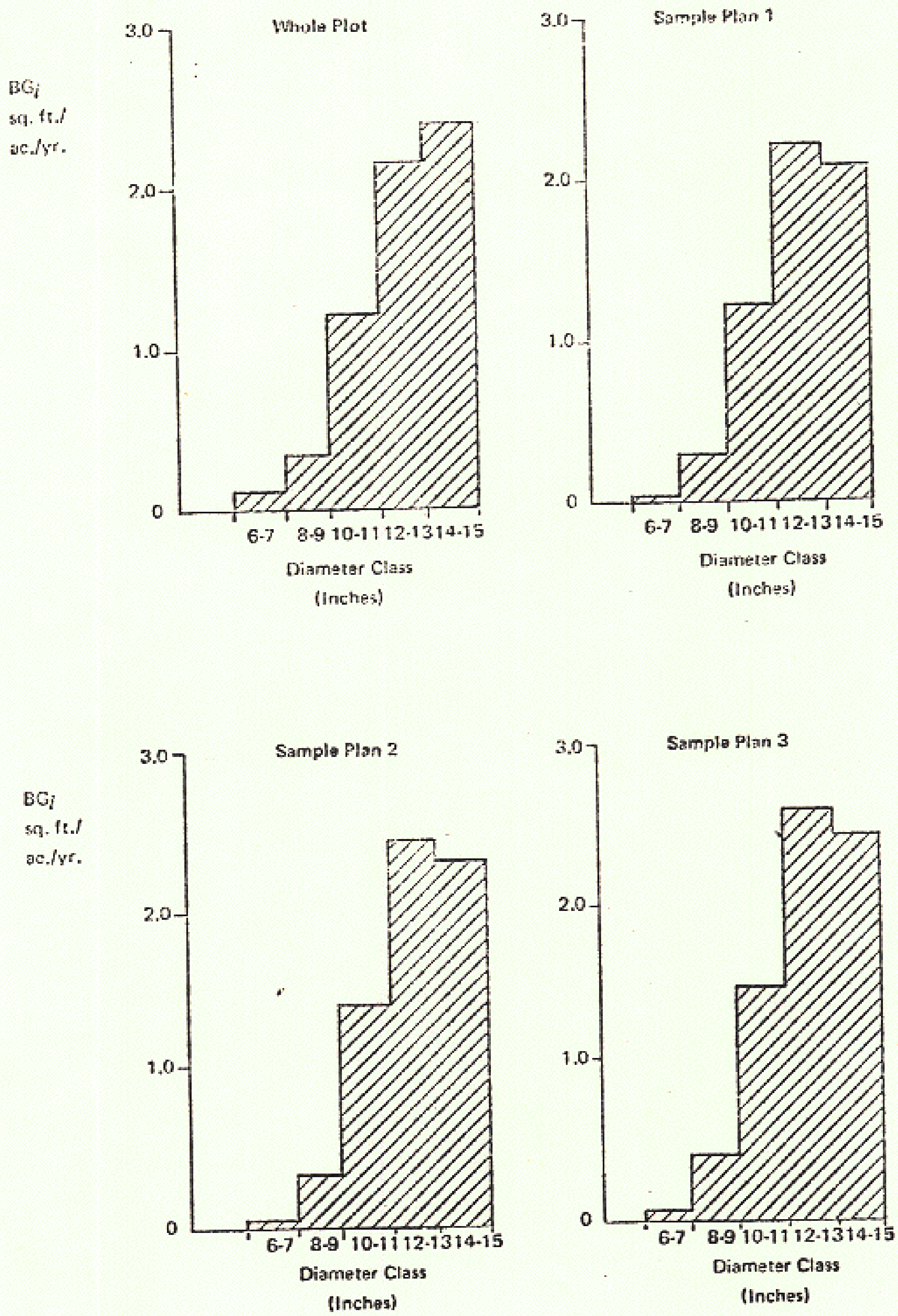

Figure 10. Comparison of estimates of basal area increment distribution (BG_i) per acre per year for sample plot 5.

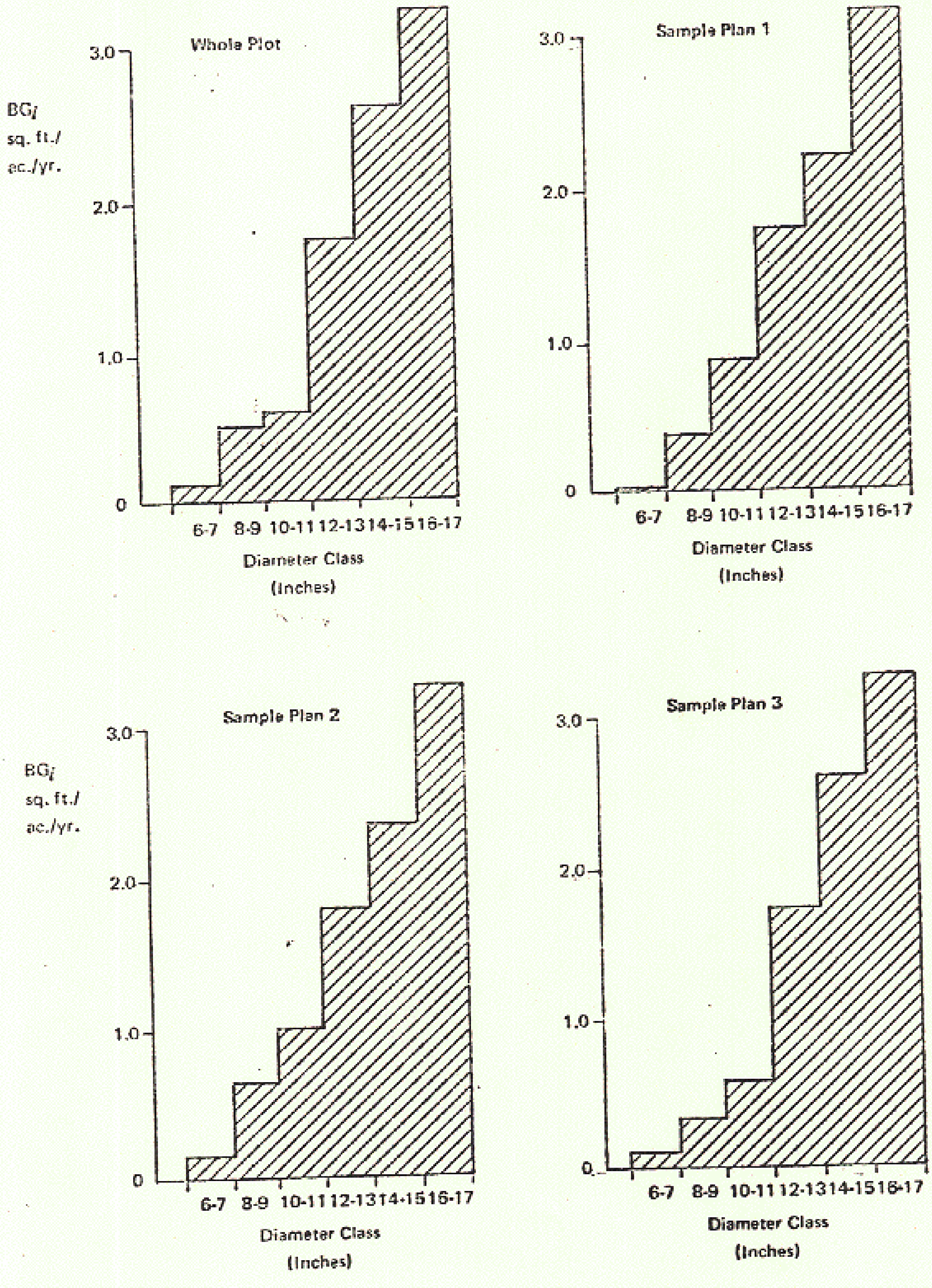

Figure 11. Comparison of estimates of basal area increment distribution (BG_i) per acre per year for sample plot 6.

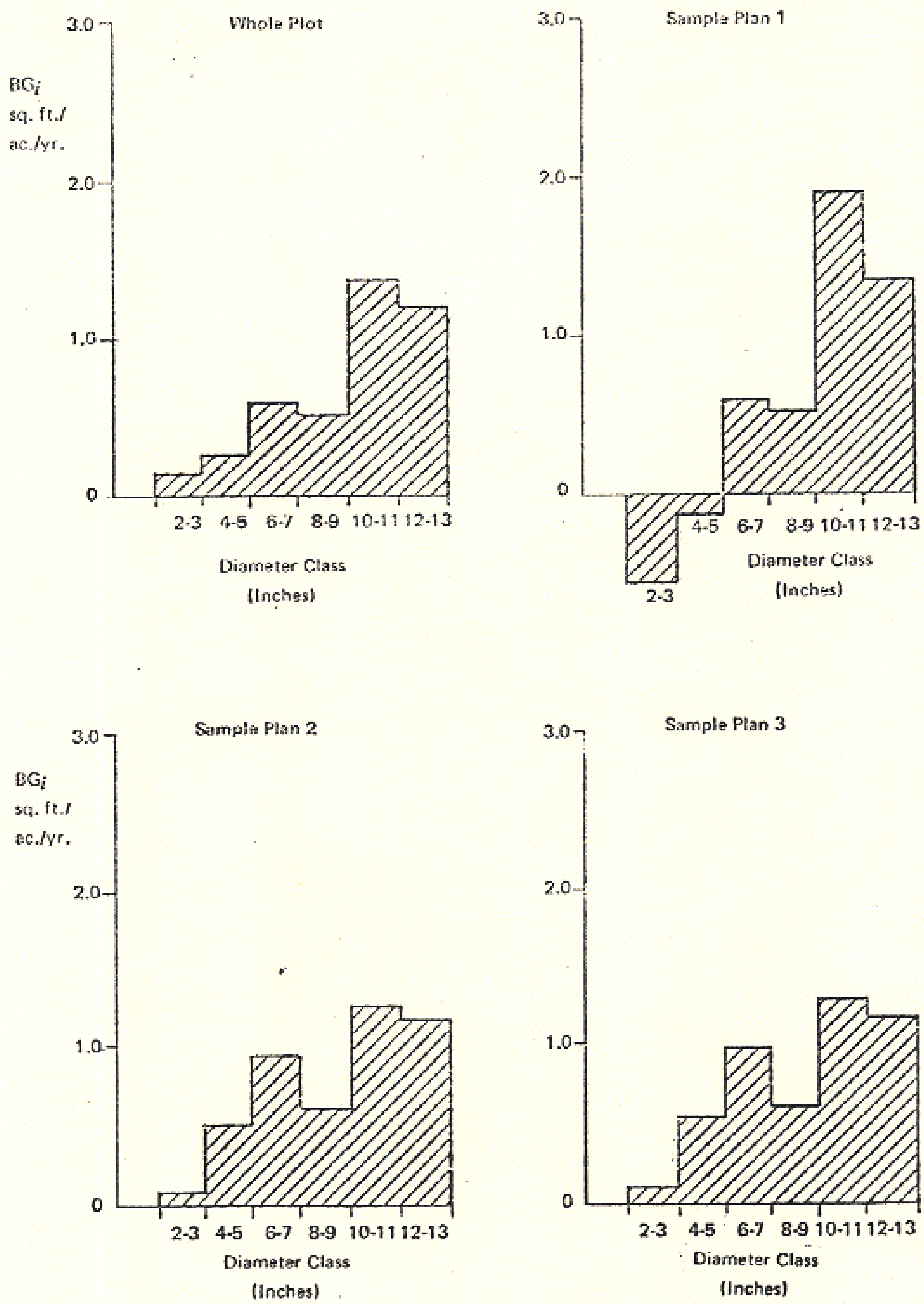

Figure 12. Comparison of estimates of basal area increment distribution [BG_i] per acre per year for sample plot 7.

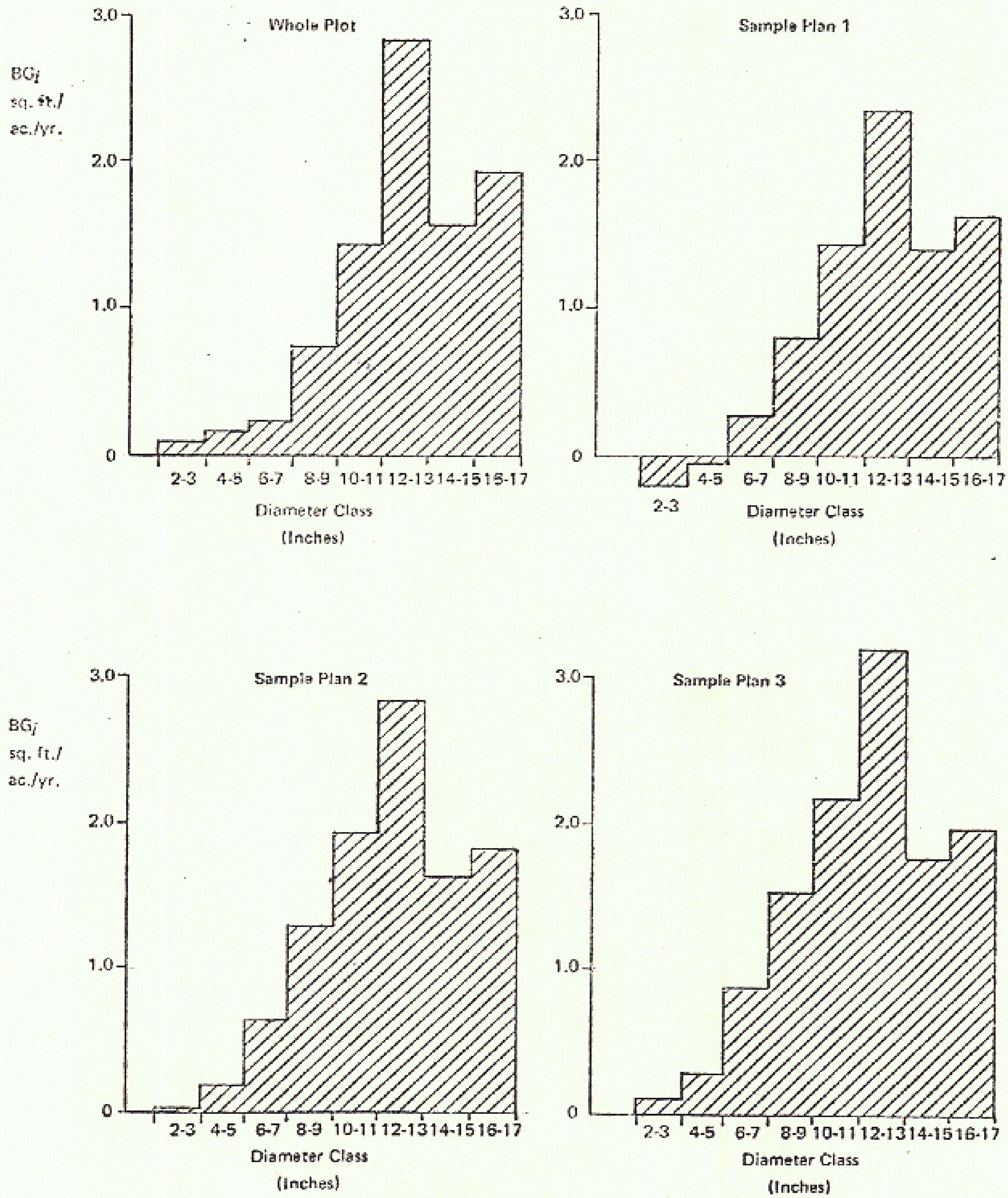

Figure 13. Comparison of estimates of basal area increment distribution [BG$_i$] per acre per year for sample plot 8.